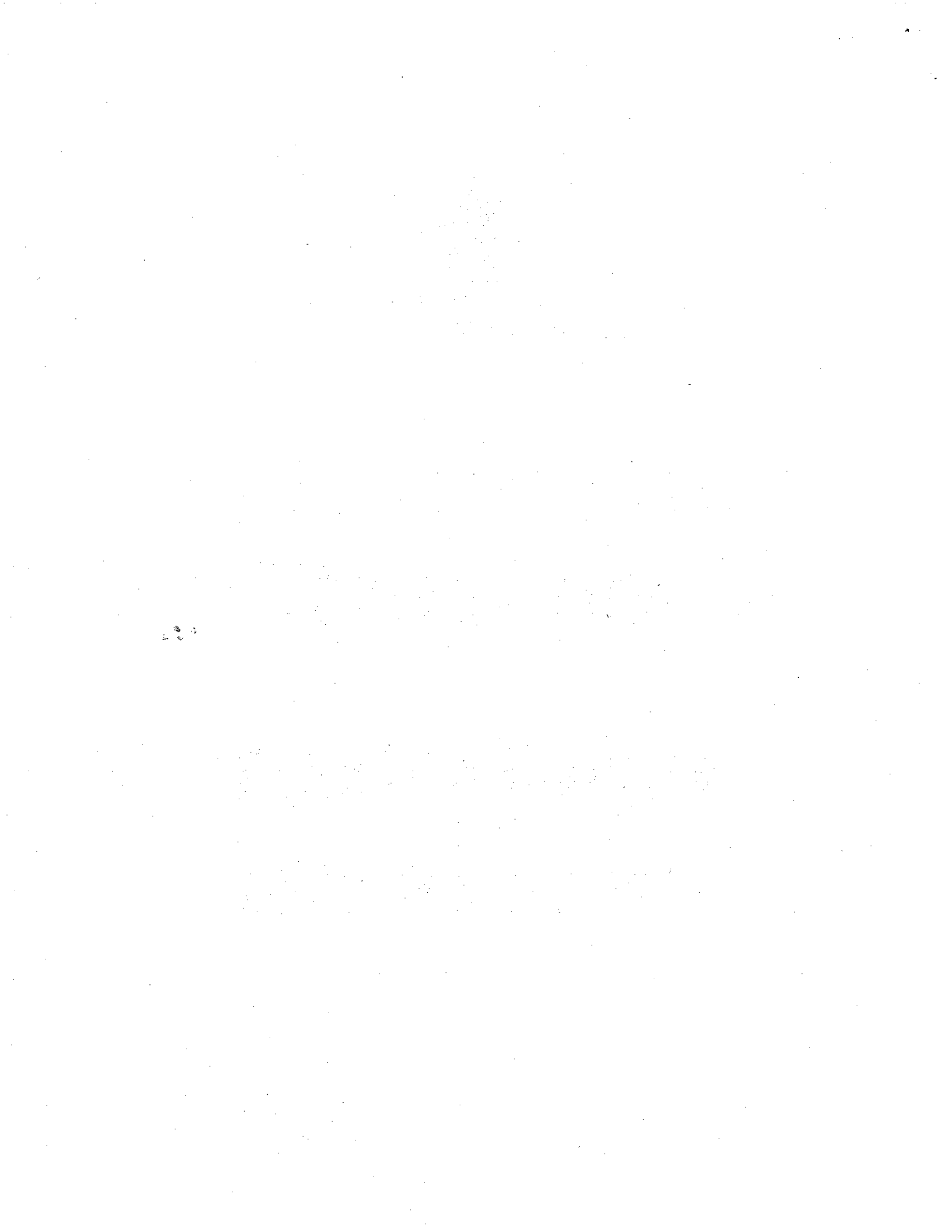


**UTAH STATE MATH
CONTEST**

**JUNIOR TEST
GRADES 7-9**



1. A merchant had a forty-pound measuring weight that broke into four pieces as the result of a fall. When the pieces were subsequently weighed, it was discovered that the weight of each piece was a whole number of pounds and that the four pieces could be used to weigh every integral weight between 1 and 40 pounds. The merchant could accomplish this by applying various combinations of the four pieces to either side of the scale. Which of the following sets contains all of the correct weights of the four pieces?
- A. {3, 4, 10, 11, 25, 26} B. {1, 2, 5, 10, 11, 20} C. {1, 4, 5, 18, 21, 29}
 D. {1, 2, 3, 9, 10, 27} E. {2, 3, 7, 8, 13, 14}
2. At a party 66 handshakes took place. Each person shook hands exactly once with each other person. How many people were at the party?
- A. 8 B. 9 C. 10 D. 11 E. 12
3. If you loose 20% from your investment in the first year, and then gain 25% in the second year, what is your net gain over the two years?
- A. -5% B. -2.5% C. 5% D. 0% E. 2.5%

4. Given the pattern in the following equalities,

$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 5^2 + 12^2 &= 13^2 \\ 7^2 + 24^2 &= 25^2 \\ 9^2 + 40^2 &= 41^2 \\ 11^2 + 60^2 &= 61^2 \end{aligned}$$

which of the sets below contains the x and y that satisfy $13^2 + x^2 = y^2$.

- A. {72, 73, 74, 75, 76} B. {82, 83, 84, 85, 86} C. {77, 78, 79, 80, 81}
 D. {87, 88, 89, 90, 91} E. {92, 93, 94, 95, 96}
5. A magic square is a matrix of the numbers with the property that each row and each column sums to the same number. Three examples of a Magic square of order 3 are given below.

4	9	2
3	5	7
8	1	6

9	5	16
17	10	3
4	15	11

7	0	5
2	4	6
3	8	1

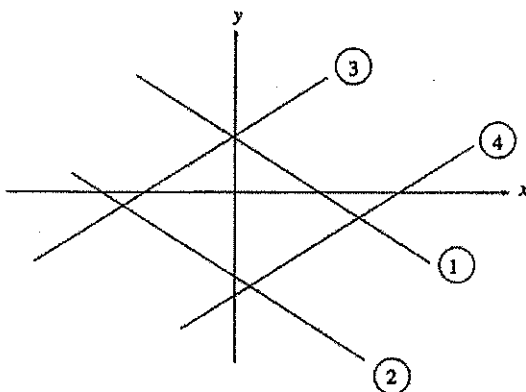
What is the value of A in the magic square below

.	93	32
.	A	35
.	.	77

- A. 72 B. 48 C. 24 D. 12 E. 32

6. Given triangle $\triangle ABC$, such that angles A , B , and C measure 50° , 60° , and 70° , respectively, which side of the triangle is the longest?
- Not enough information to determine.
 - \overline{AC}
 - \overline{BC}
 - \overline{AB}
 - All are equal.

7. If $a \cdot b > 0$, which line(s) may represent the graph of $\frac{x}{a} + \frac{y}{b} = 3$?



- 2 and 3 B. 3 and 4 C. 1 only D. 1 and 2 E. 4 only
8. The polynomial $f(x) = ax^2 + bx + c$ has two real distinct roots. The distance between these two roots is d . What is the distance between the roots of $f(2x)$?
- d B. $\frac{1}{4}d$ C. $2d$ D. \sqrt{d} E. $\frac{1}{2}d$
9. What is the least common multiple of the first 10 counting numbers?
- 10080 B. 840 C. 1260 D. 2520 E. 5040
10. Solve for x
- $$x + \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}} = 2$$
- $\pm\sqrt{2}$ B. $2 \pm \sqrt{2}$ C. $\sqrt[3]{2}$ D. $\sqrt[4]{2}$ E. $2 - \sqrt{2}$
11. Fifteen percent of the students in a classroom like to watch hockey games. Twenty five percent of the students in the classroom like to watch baseball games. Among all of the students who like to watch hockey games, sixty percent also like to watch baseball games. What is the percentage of the students who like to watch neither game?
- 40% B. 50% C. 69% D. 31% E. 64%

12. Which set contains positive integer values for x , y , and z satisfying

$$x + \frac{y}{19} + \frac{z}{97} = \frac{1997}{19 \times 97}$$

- A. $\{1, 2, 4\}$ B. $\{3, 8, 11\}$ C. $\{1, 3, 5\}$ D. $\{5, 7, 9\}$ E. $\{8, 11, 13\}$

13. Which set contains positive integers u and v satisfying the following equality?

$$\sqrt{18 - 2\sqrt{65}} = \sqrt{u} - \sqrt{v}$$

- A. $\{3, 5, 7, 9, 11, 13\}$ B. $\{6, 9, 12, 15, 18, 21\}$ C. $\{2, 4, 6, 8, 10, 12\}$ D. $\{1, 4, 7, 10, 13, 16\}$
E. $\{5, 10, 15, 20, 25, 30\}$

14. One ticket is drawn at random from Box A and one from Box B. Box A has three tickets with the numbers 4,9,9 written on them. Box B has four tickets marked 4,5,7,9. You will win \$5 if the number from Box A is the same as Box B. Otherwise, you win nothing and lose nothing.

What is the chance to win \$5?

- A. $\frac{1}{3}$ B. $\frac{1}{6}$ C. $\frac{1}{4}$ D. $\frac{5}{6}$ E. $\frac{1}{2}$

15. A soft drink manufacturer has selected a random sample of consumers whose ages are in the range 18-40 years. The people in the sample were asked to provide the beverage that they consume most often. The participants were classified by their age and their beverage choice. The results are summarized in the contingency table below:

Age	Soft Drink(S)	Coffee (C)	Tea(T)	Total
18-25 years (A)	400	50	10	460
26-40 years (B)	180	150	110	440
Total	580	200	120	900

What is the probability that a randomly selected consumer will have Tea (T) as his beverage preference and is NOT in the range 26-40 years (B)?

- A. $\left(\frac{120}{900}\right) \left(\frac{460}{900}\right)$ B. $\left(\frac{10}{120}\right) \left(\frac{10}{460}\right)$ C. $\frac{10}{900}$ D. $\frac{10}{460}$ E. $\frac{120}{900} + \frac{460}{900} - \frac{10}{900}$

16. Which set contains all the integers making $f(x)$ a perfect square?

$$f(x) = x^2 - 7x - 4$$

Note: some examples of a perfect square are 16, 25, 36.

- A. $\{-20, -4, 2, 14, 23, 35\}$ B. $\{-21, -12, 7, 19, 28, 40\}$ C. $\{-16, -3, 9, 18, 21, 30\}$
D. $\{-15, -13, -1, 8, 11, 20\}$ E. $\{-17, -8, 4, 13, 25, 26\}$

17. Express $1.737373737\dots$ as a fraction.

- A. $\frac{172}{99}$ B. $\frac{173}{101}$ C. $\frac{173}{98}$ D. $\frac{173}{100}$ E. $\frac{73}{100}$

18. Hank has a pocket full of pennies, nickles, dimes, and quarters, with at least one of each coin. He has a total of 19 coins worth 93 cents. How many dimes does he have?

- A. 2 B. 3 C. 4 D. 5 E. 6

19. Factor completely $6m^{n+2} - 8m^{n+1} - 9m^2 + 12m$

- A. $m(3m^n + 3)(2m - 4)$ B. $m(2m^n - 3)(3m + 4)$ C. $m(3m^n - 3)(3m - 4)$
 D. $m(3m^n - 3)(2m + 4)$ E. $m(2m^n - 3)(3m - 4)$

20. Find an equivalent expression for $(((((x^5)^0)^9)^{11})^{13})$, where $x \neq 0$.

- A. 1 B. x^{45045} C. x^{450} D. x^{45} E. 0

21. A princess is four years older than a prince. Moreover, the princess is as old as the prince will be when the princess is twice the age of the prince when the princess is half the sum of their current ages. How old is the princess?

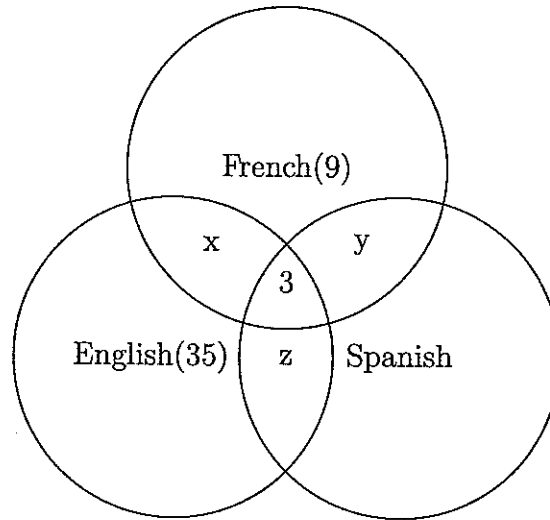
- A. 34 B. 16 C. 22 D. 20 E. 18

22. Ginny has 15 cans of soup in her food pantry: 7 vegetable lentil, 3 split pea, and 5 minestrone. Ginny will be very busy for the next three days, so she is planning to serve everyone in her family a can of soup for the next three dinners. Ginny has a family of three and she will randomly pick three cans of soup for each dinner. For the **first dinner**, what is the chance that all family members will have the same type of soup?

Note: $\binom{m}{p}$ represents the number of ways of selecting p items from a collection of m items.

- A. $\frac{\binom{3}{7} + \binom{3}{5} + \binom{3}{3}}{3}$ B. $\binom{7}{3} + \binom{3}{3} + \binom{5}{3}$ C. $\frac{\binom{7}{3} + \binom{3}{3} + \binom{5}{3}}{\binom{15}{3}}$
 D. $\frac{\binom{7}{3} + \binom{3}{2} + \binom{5}{1}}{\binom{5}{3}}$ E. $\binom{7}{3} + \binom{3}{2} + \binom{5}{1}$

23. A survey showed that among a certain set of 50 people, only 3 spoke all three languages, English, Spanish, and French, while 18 spoke exactly two of these languages and 25 spoke just one of these languages. A total of 35 people spoke English. Also a total of 9 spoke French, but each of these also spoke English or Spanish, or both. How many of the 50 people spoke Spanish. The following Venn diagram might help in setting up a system of equations.



- A. 16 B. 26 C. 18. D. 24 E. 28
24. John is taking a quiz that has two multiple-choice questions. Each question consists of four choices (A, B, C, and D) and has only one correct answer. Suppose that he did not study for the quiz and had to randomly guess the answer to each question; as such, each answer is selected independently of the other answer. What is the probability that he will have **only one** correct answer if he answers both questions.
- A. $\frac{3}{16}$ B. $\frac{1}{2}$ C. $\frac{9}{256}$ D. $\frac{3}{8}$ E. $\frac{3}{8}$

25. Twelve cartons of eggs are donated to a food bank. Unknown to anyone, there are five of them that are contaminated. There are five people in a line waiting to get a free carton of eggs. What is the probability that the third person will be the first person to get a contaminated carton of eggs?

Note: $\binom{m}{p}$ represents the number of ways of selecting p things from a collection of m total things.

A. $\frac{7}{12} \cdot \frac{6}{11} \cdot \frac{5}{10}$ B. $\frac{1}{12} \cdot \frac{1}{11} \cdot \frac{5}{10}$ C. $\frac{\binom{7}{2}\binom{5}{1}}{\binom{12}{3}}$ D. $\frac{\binom{7}{2}\binom{5}{1}\binom{12}{3}}{\binom{12}{5}}$

E. $\frac{\binom{12}{2}\binom{5}{3}}{\binom{12}{3}}$

26. A math professor posts the grade distribution for his algebra course as shown below.

Grade	A	B	C	D	F
# Students	11	34	42	9	4

If the professor selects two different students at random, what is the probability that both students will receive grade B or above?

Hint: There are four possible events that both students will receive grade B or above:

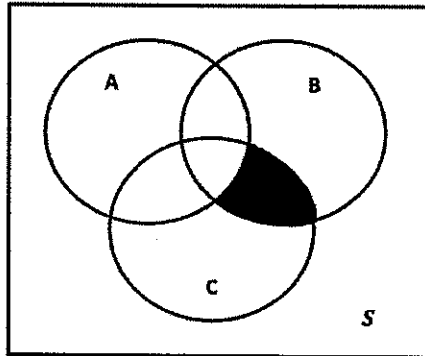
1. The first student will receive grade A and the second student will receive grade A.
2. The first student will receive grade A and the second student will receive grade B.
3. The first student will receive grade B and the second student will receive grade A.
4. The first student will receive grade B and the second student will receive grade B.

Note: $\binom{m}{p}$ represents the number of ways of selecting p things from a collection of m total things.

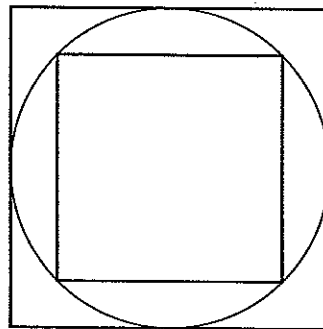
A. $\frac{\binom{45}{2}}{\binom{100}{2}}$ B. $\frac{11}{100} \binom{34}{99}$ C. $\frac{(45+44)}{(100+99)}$ D. $\frac{\binom{45}{2}}{\binom{100}{2}}$ E. $\frac{\binom{99}{34}}{\binom{100}{2}}$

27. Given the Venn diagram that represents the events A, B, and C. What is the set represented by the shaded region?

Note: The superscript c denotes complement. The notation \cup denotes union. The notation \cap denotes intersection.



- A. $A^c \cup (B \cap C)$ B. $A^c \cap B \cap C$ C. $A^c \cup (B \cup C)$ D. $(A \cup B)^c \cap C$
 E. $(C \cap A) \cup (B \cap A)^c$
28. given a circle of radius 1 inscribed inside of a square, and then a square inscribed inside the circle, what is the area enclosed by the two squares?



- A. $\sqrt{2}$ B. $2 - \frac{1}{\sqrt{2}}$ C. $4 - \sqrt{2}$ D. 2 E. 4
29. Which set contains the three distinct odd positive integers , a, b, and c such that

$$\frac{1}{3} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

- A. {15, 21, 29, 35} B. {11, 19, 23, 27} C. {9, 17, 23, 33}
 D. {7, 13, 19, 33} E. {5, 9, 11, 45}
30. Suppose the two quadratic equations $x^2 - 5x + k = 0$ and $x^2 - 9x + 3k = 0$ have a nonzero root in common. What set contains the value of k ?
- A. {6, 11, 16, 21} B. {2, 7, 12, 17} C. {3, 8, 13, 18} D. {4, 9, 14, 19}
 E. {5, 10, 15, 20}

