State Math Contest 2018 Junior Exam

Weber State University

March 8, 2018

Instructions:

- Do not turn this page until your proctor tells you.

- Enter your name, grade, and school information following the instructions given by your proctor.

- Calculators are not allowed on this exam.

- This is a multiple choice test with 40 questions. Each question is followed by answers marked a), b), c), d), and e). Only one answer is correct.

- Mark your answer to each problem on the bubble sheet answer form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.

- Scoring: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.

- You will have 2 hours and 30 minutes to finish the test.

- You may not leave the room until at least 10:15 a.m.

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1. Zoey read 15 books, one at a time. The first book took her one day to read, the second book took her 2 days to read, the third book took her 3 days to read, and so on, with each book taking her 1 more day to read than the previous book. Zoey finished the first book on a Monday, and the second on a Wednesday. On what day the week did she finish her 15th book?

   a) Sunday   b) Monday   c) Wednesday

   d) Friday   e) Saturday

2. There are four novels that you want to read over the summer. Two of the novels are by the same author and you don't want to read those directly after one another. In how many orders can you read the four novels?

   a) 4   b) 6   c) 12

   d) 18   e) 24

3. Hans Larsen flew his plane 400 km with the wind in the same time it took him to fly 200 km against the wind. The speed of the wind is 50 km per hour. Find the speed of the plane in still air.

   a) 120 km/hr   b) 150 km/hr   c) 165 km/hr

   d) 180 km/hr   e) 200 km/hr

4. If two fair dice are rolled, what is the probability that the sum of the two dice is at least 4?

   a) $\frac{1}{12}$ b) $\frac{5}{36}$ c) $\frac{11}{12}$

   d) $\frac{1}{6}$ e) $\frac{4}{12}$

5. Given the point $A = (-2, 6)$ and the line $y = 2x$, what are the coordinates of the point B obtained by reflecting A over the line $y = 2x$?

   a) $B = (7, 2)$ b) $B = (6, 3)$ c) $B = (5, 2.5)$

   d) $B = (7, 1.5)$ e) $B = (6, 2)$

6. The equation for the directrix of a certain parabola in the xy-plane is given by $y = -1$. If the vertex of the parabola is at $(7, 3)$ what is the equation of the parabola?

a) $16(y - 7) = (x - 3)^2$

b) $16(y - 3) = (x - 7)^2$

c) $12(y - 4) = (x - 7)^2$

d) $14(y - 3) = (x - 4)^2$

e) $4(y - 3) = (x - 4)^2$

7. A cold-water faucet can fill a sink in 12 minutes, and a hot-water faucet can fill the same sink in 15 minutes. The drain at the bottom of the sink can empty the sink in 25 minutes. If both faucets and the drain are open, how long will it take to fill the sink?

a) $5\frac{15}{57}$ minutes

b) $7\frac{3}{4}$ minutes

c) $9\frac{1}{11}$ minutes

d) $10\frac{4}{11}$ minutes

e) $20\frac{4}{57}$ minutes

8. How many ways can you write 7 as the sum of one or more positive integers if different orders are not counted differently? For example, there are three ways to write 3 in this way: 3, $2 + 1$, and $1 + 1 + 1$.

a) 6

b) 7

c) 13

d) 14

e) 15

9. What is the area of $\triangle ABC$ shown below where $AB = BC$, $AC = 2$ inches, and $\angle BCD = 120°$?
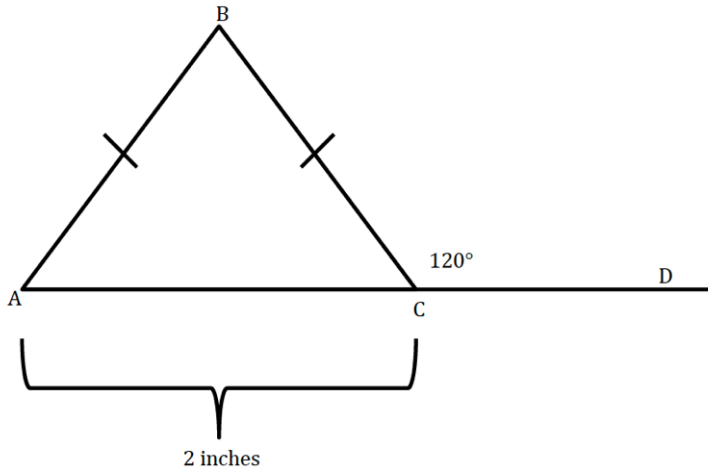
a) $\sqrt{3}$ in^2

b) $2\sqrt{3}$ in^2

c) 1 in^2

d) 2 in^2

e) 4 in^2

10. Ryan, Eric, and Kim like to hike mountains. Kim hiked six mountains that neither Ryan nor Eric hiked. There were only four mountains that all three hikers climbed. There was only one mountain Eric and Ryan hiked that Kim did not hike. There were no mountains that only Eric and Kim hiked. Ryan hiked six times more mountains than Eric. Kim hiked $\frac{1}{3}$ of the mountains that Ryan hiked and two times more mountains that Eric hiked. How many mountains did only Ryan hike?

    a) 22   b) 27   c) 29

    d) 48   e) 56

11. Find the sum of the first 100 terms for the following sequence of numbers 35, 37, 39, ...

    a) 13,400   b) 27,000   c) 13,500

    d) 26,800   e) 13,600

12. What is the smallest integer that is one-half of a square and one-third of a cube?

    a) 64   b) 1728   c) 72

    d) 36   e) 2

13. If you divide John's age by 5, its remainder is 1. And if you divide John's age by 9, its remainder is 7. How old is Johnny today, if he had a secondary school math exam yesterday, on his birthday?

    a) 12   b) 14   c) 16

    d) 18   e) 20

14. How many positive whole numbers (or positive integers) less than or equal to 30 have either 2 or 3 as a factor?

    a) 5   b) 10   c) 15

    d) 20   e) 25

15. The length of all three sides of a triangle are integer numbers. The length of one side is 6 and the length of the second side is 1. What is the length of the third side of the triangle?

    a) 1   b) 3   c) 5

    d) 6   e) 7

16. What is the difference between the sum of the first 500 positive even numbers and the first 500 positive odd numbers?

    a) 5   b) 10   c) 100

    d) 500   e) 1000

17. If $f(x - 1) = (1 - x)(x + 2)(x - 3)$, on which intervals is the function $f(x)$ negative?

    a) $(-2, 1)$ b) $(-2, 1) \cup (2, \infty)$ c) $(-\infty, -3) \cup (-2, 1)$

    d) $(-3, 0) \cup (2, \infty)$ e) None of the above are possible.

18. Solve $\log_4 (x^2 - 9) - \log_4 (x + 3) = 3$

    a) -67 b) 67 c) 3

    d) -3 e) None of the above are possible.

19. Find the value of $\frac{a}{b}$ if $\frac{a}{b} = \frac{a+48}{b+6}$.

    a) 24   b) 144   c) 8

    d) 6   e) 48

20. A family with four children has a girl named Nancy. What is the probability that all four children are girls, if the probability of having a boy is $\frac{1}{2}$ and the probability of having a girl is $\frac{1}{2}$?

    a) 1   b) $\frac{1}{2}$ c) $\frac{1}{16}$

    d) $\frac{1}{15}$ e) $\frac{1}{4}$

21. Find x if $25^{(x+1)} = 625^{(2x)}$.

    a) $x = \frac{1}{4}$ b) $x = \frac{1}{3}$ c) $x = \frac{1}{2}$

    d) $x = \frac{2}{3}$ e) $x = \frac{3}{4}$

22. On level ground, a 10-foot pole is a certain distance from a 15-foot pole. If lines are drawn from the top of each pole to the bottom of the other pole as shown in the drawing, the lines intersect at a point 6 feet above the ground. What is the distance between the poles?

    a) 12.5 feet   b) 5 feet   c) 25 feet

    d) None of the above are possible.   e) Any of the above are possible.

23. Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

    a) 30%   b) 40%   c) 50%
    d) 60%   e) 70%

24. Find the volume of a cone whose base has an area of two square-units and height of six units.

    a) 4   b) 6   c) 2

    d) 2.33   e) 4.33

25. Find the smallest positive integer that is divisible by exactly 11 positive integers.

    a) 72   b) 108   c) 576

    d) 500   e) 1024

26. If $a > 0$, then the quadratic function $y = x^2 + ax + a$ has

    a) One zero   b) Two zeros   c) No zeros

    d) Any of the above are possible.   e) None of the above are possible.

27. A rectangle is partitioned into 4 subrectangles as shown below. If the subrectangles have the indicated areas, find the area of the unknown rectangle.

210	240
91	?

a) 78   b) 98   c) 104

d) 270   e) 390

28. The total number of passengers riding a certain city bus during the during the day is 1,000. If the child?s fare is $0.50, the adult fare is $1, and the senior fare is $0.50, and the total revenue from the day is $600. It is also known that an equal number of seniors ride the bus as children. Use this information to determine total number of children and adults that ride the bus.

a) 600   b) 200   c) 300

d) 133   e) 400

29. You and two of your friends go to a restaurant for lunch. At the end of the meal, all three of you flip a fair coin, getting either a head or a tail. If two of the three coin flip outcomes match, the person with the non-matching outcome pays for everyone's lunch. If all three coin flip outcomes are the same, each person pays for their own lunch. What is the probability that you pay for everyone's lunch?

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$

d) $\frac{1}{6}$ e) $\frac{1}{8}$

30. A rain gutter is to be made of aluminum sheets that are 12 inches wide by turning up the edges 90°. What depth will provide maximal cross-sectional area and hence allow the largest water flow?

a) 4.5 inches   b) 2 inches   c) 3.5 inches

d) 4 inches   e) 3 inches

31. A group of friends took a bus trip. Each traveler gave the bus driver a tip using the same nine coins. The total tip was \$8.41. How many dimes did the driver receive?

    a) 0   b) 9   c) 18

    d) 27   e) 36

32. Seven friends took a quiz. Each got a score that is a whole number between 1 and 100. No two friends got the same score. The median score received by the friends was 50 and the range (the maximum score minus the minimum score) was 20. What is the highest score that any of the friends could have received?

    a) 50   b) 53   c) 60

    d) 67   e) 99

33. Pablo buys popsicles for his friends. The store sells single popsicles for \$1 each, 3-popsicle boxes for \$2 each, and 5-popsicle boxes for \$3. What is the greatest number of popsicles that Pablo can buy with \$8?

    a) 8   b) 11   c) 12

    d) 13   e) 15

34. Find the last digit of 3^{999}.

    a) 1   b) 3   c) 7

    d) 9   e) None of the above are possible.

35. What is the domain of the function $f(x) = (x - 3x^2)^{\frac{1}{2}}$?

    a) $[0, \frac{1}{3}]$ b) $(-\infty, \frac{1}{3}]$ c) $(-\infty, 0) \cup (\frac{1}{3}, \infty)$

    d) $(0, \frac{1}{3})$ e) $(-\infty, \infty)$

36. The Golden Gate Bridge is a suspension bridge which spans the entrance to San Francisco Bay. Its 720 foot tall towers are 4000 feet apart. The bridge is suspended from two huge cables. The roadway is 220 feet above the base of the towers. The cables are parabolic in shape and touch the road surface at the center of the bridge. Find the height of the cable above the road at a distance of 1000 feet from the center of the bridge.

4000 ft.

720 ft.

220 ft.
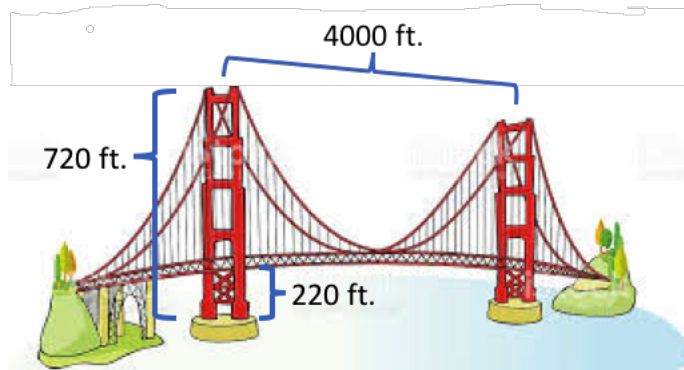
    a) 360 feet   b) 250 feet   c) 150 feet

    d) 125 feet   e) 180 feet

37. Find all the values of x which satisfy the inequality $\sqrt{(x-2)^2} < |x|$.

    a) $(-2,0)$ b) $(-2,1) \cup (2,\infty)$ c) $(1,\infty)$

    d) $(2,5)$ e) None of the above are possible.

38. $(3\sqrt{7}+4)^3 + (3\sqrt{7}-4)^3 =$

    a) 1   b) $3\sqrt{7}$ c) $21\sqrt{7}$

    d) $111\sqrt{7}$ e) $666\sqrt{7}$

39. Let x and y have the following relationship:

$$3xy^2 - 3yx^2 = 6$$
$$x^3 - y^3 = -7$$

Determine the value of $(x-y)^3 =$

    a) -1 b) 1   c) 0

    d) -8 e) 27

40. Find the difference in area of the two circular discs:

$$(x + 3)^2 + (y - 6)^2 = 4$$
$$(x + 3)^2 + (y - 5)^2 = 9$$

a) 4π b) 5π c) 7π

d) 9π e) 11π