UTAH STATE MATH CONTEST

SENIOR TEST

GRADES 10-12

1. A merchant had a forty-pound measuring weight that broke into four pieces as the result of a fall. When the pieces were subsequently weighed, it was discovered that the weight of each piece was a whole number of pounds and that the four pieces could be used to weigh every integral weight between 1 and 40 pounds. Which of the following sets contains all of the correct weights of the four pieces?

   A. (3,4, 10, 11, 25, 26, 31) B. (1,2,5,10,11, 20,21) C. (1,4,5,18,21,29, 39)
   D. (1,2,3,9,10, 27, 28) E. (2,3,7,8, 13,14, 31)

2. At a party 66 handshakes took place. Each person shook hands exactly once with each other person. How many people were at the party?

   A. 8 B. 9 C. 10 D. 11 E. 12

3. A palindrome can be defined as a number that may be read the same way in either direction. What is the maximum number of six digit palindromes?

   A. 100 B. 1440 C. 720 D. 1000 E. 600

4. A magic square is a matrix of numbers with the property that each row and each column sums to the same number. Three examples of a Magic square of order 3 are given below.

4	9	2
3	5	7
8	1	6

9	5	16
17	10	3
4	15	11

7	0	5
2	4	6
3	8	1

   What is the value of A in the magic square below

·	93	32
·	A	35
·	·	77

   A. 72 B. 48 C. 24 D. 12 E. 32

5. The midpoints of the three sides of $\triangle ABC$ are $(5,4)$, $(1,2)$, and $(-1,6)$. Which of the following is one of the actual points A, B, or C?

   A. $(4,9)$ B. $(3,8)$ C. $(0,6)$ D. $(5,2)$ E. $(6,3)$

6. What is the sum of the first 40 even integers?

   A. 400 B. 440 C. 880 D. 1640 E. 1320

7. Which set contains positive integer values for x, y, and z satisfying

$$x + \frac{y}{19} + \frac{z}{97} = \frac{1997}{19 \times 97}$$

   A. $\{1,2,4\}$ B. $\{3,8,11\}$ C. $\{5,7,9\}$ D. $\{1,3,5\}$ E. $\{8,11,13\}$

8. Which set contains positive integers u and v satisfying the following equality?

$$\sqrt{18-2\sqrt{65}} = \sqrt{u} - \sqrt{v}$$

A. $\{6,9,12,15,18,21\}$ B. $\{3,5,7,9,11,13\}$ C. $\{2,4,6,8,10,12\}$ D. $\{1,4,7,10,13,16\}$ E. $\{5,10,15,20,25,30\}$

9. Suppose the two quadratic equations $x^2-5x+k=0$ and $x^2-9x+3k=0$ have a nonzero root in common. What set contains the value of k?

A. $\{2,7,12,17\}$ B. $\{3,8,13,18\}$ C. $\{4,9,14,19\}$ D. $\{6,11,16,21\}$ E. $\{5,10,15,20\}$

10. Which set contains all the integers making $f(x)$ a perfect square?

$$f(x) = x^2 - 7x - 4$$

Note: some examples of a perfect square are 16, 25, 36.

A. $\{-20,-4,2,14,23,35\}$ B. $\{-21,-12,7,19,28,40\}$ C. $\{-16,-3,9,18,21,30\}$ D. $\{-17,-8,4,13,25,26\}$ E. $\{-15,-13,-1,8,11,20\}$

11. Express $1.737373737\ldots$ as a fraction.

A. $\frac{173}{101}$ B. $\frac{173}{98}$ C. $\frac{172}{99}$ D. $\frac{173}{100}$ E. $\frac{73}{100}$

12. A math professor posts the grade distribution for his algebra course as shown below.

Grade	A	B	C	D	F
# Students	11	34	42	9	4

If the professor selects two different students at random, what is the probability that both students will receive grade B or above?

A. $\frac{11}{100}\binom{34}{99}$ B. $\frac{(45+44)}{(100+99)}$ C. $\frac{\binom{45}{2}}{\binom{100}{2}}$ D. $\frac{\binom{45}{2}}{\binom{100}{2}}$ E. $\frac{\binom{99}{34}}{\binom{100}{2}}$

13. Sum the series $\sum_{n=1}^{\infty}(\frac{-2}{3})^n$.

A. $-\frac{2}{5}$ B. $\frac{2}{5}$ C. $\frac{3}{2}$ D. $-\frac{3}{2}$ E. The series diverges

14. How many minutes after 7:00pm will the minute hand of a watch overtake the hour hand?

A. $37\frac{5}{11}$ B. $35\frac{3}{11}$ C. $38\frac{3}{11}$ D. $37\frac{5}{11}$ E. $38\frac{4}{11}$

15. The expression $\sin 2x + (\cos x - \sin x)^2$ is equivalent to

A. $\sin^2 x - 1$ B. -1 C. 1 D. $2\sin x \cos x$ E. $\cos^{2x} - \sin^{2x}$

16. Hank has a pocket full of pennies, nickles, dimes, and quarters, with at least one of each coin. He has a total of 19 coins worth 93 cents. How many dimes does he have?

A. 5 B. 4 C. 3 D. 2 E. 1

17. Using the fact that

$$e^{(x-e)/e} \geq 1 + \frac{x-e}{e}$$

what positive value of x will maximize the expression $\sqrt[x]{x}$?

A. $x = e$ B. $x = \sqrt{e}$ C. $x = e^e$ D. $x = 1/e$ E. $x = e!$

18. A princess is four years older than a prince. Moreover, the princess is as old as the prince will be when the princess is twice the age of the prince when the princess is half the sum of their current ages. How old is the princess?

A. 16 B. 34 C. 22 D. 20 E. 18

19. Given the pattern in the following equalities,

$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 5^2 + 12^2 &= 13^2 \\ 7^2 + 24^2 &= 25^2 \\ 9^2 + 40^2 &= 41^2 \\ 11^2 + 60^2 &= 61^2 \end{aligned}$$

which of the sets below contains the x and y that satisfy $13^2 + x^2 = y^2$.

A. {72, 73, 74, 75, 76} B. {77, 78, 79, 80, 81} C. {87, 88, 89, 90, 91}
D. {92, 93, 94, 95, 96} E. {82, 83, 84, 85, 86}

20. Which of the following sets contains the minimum value of the numbers in the sequence:

$$\left(\sqrt{\frac{7}{6}} + \sqrt{\frac{96}{7}}\right), \left(\sqrt{\frac{8}{6}} + \sqrt{\frac{96}{8}}\right), \left(\sqrt{\frac{9}{6}} + \sqrt{\frac{96}{9}}\right), \left(\sqrt{\frac{10}{6}} + \sqrt{\frac{96}{10}}\right), \cdots, \left(\sqrt{\frac{95}{6}} + \sqrt{\frac{96}{95}}\right),$$

Hint: there are several ways to solve this problem - these tips might help.

$$23^2 = 529,\ 24^2 = 576,\ 25^2 = 625,\ 26^2 = 676.$$

and for any $a, b \geq 0$ $a + b \geq 2\sqrt{ab}$.

A. {4, 8, 16, 32} B. {5, 10, 15, 20} C. {6, 18, 36, 72} D. {7, 21, 28, 35 }
E. {3, 9, 27, 81}

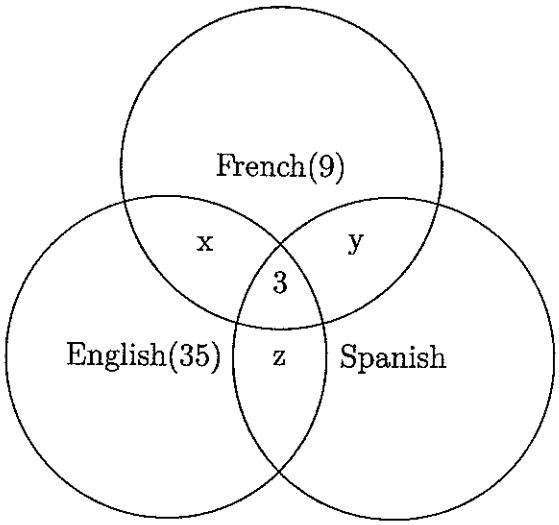
21. If a function, $f(x)$, has the property that

$$|f(a) - f(b)| \leq |a - b|^2$$

for any real numbers a and b, then the function $f(x)$ must be a

A. parabola. B. a constant. C. $\sin(x)$. D. upper half of a semi-circle.
E. square root of x.

22. A survey showed that among a certain set of 50 people, only 3 spoke all three languages, English, Spanish, and French, while 18 spoke exactly two of these languages and 25 spoke just one of these languages. A total of 35 people spoke English. Also a total of 9 spoke French, but each of these also spoke English or Spanish, or both. How many of the 50 people spoke Spanish. The following Venn diagram might help in setting up a system of equations.



A. 16 B. 18 C. 26. D. 24 E. 28

23. A researcher took a random sampling of 20 students and asked the amount of cash they had in their wallets. He found that the contents of the students wallets had a mean of \$42, as well as a median of \$42. There were no perceived outliers in the group. At this point, the researcher realized he had left a 21st student off of his calculations. The 21st student had \$80. After recalculating statistics for all 21 students, which of the following statements **must** be true?

I The mean increased

II The median increased

III The range increased

IV The mean is greater than the median

A. I & IV B. I C. I, II, & IV D. I, III, & IV E. I, II, III & IV

24. What is the probability that a randomly selected 10-digit number contains all ten different digits?

A. $\frac{9 \cdot 9!}{10^{10}}$ B. $\frac{10! - 9!}{10^9 - 1}$ C. $\frac{9!}{9 \cdot 10^9}$ D. $\frac{9!}{10^9}$ E. $\frac{10! - 9!}{10^{10} - 1}$

25. During a Christmas event, a local pet store donates the following dog food to a local animal shelter:

- 2 bags of Brand A
- 3 bags of Brand B
- 3 bags of Brand C.

The shelter has only 5 dogs total. There are 2 dogs allergic to Brand A, 2 dogs allergic to Brand B, and 1 dog allergic to Brand C. A donated bag of dog food is given to a dog in the shelter at random. What is the probability that a dog in the shelter will have allergic reaction to the food given to him?

A. $\frac{13}{40}$ B. $\frac{1}{5}$ C. $\frac{4}{5}$ D. $\frac{1}{40}$ E. $\frac{1}{8}$

26. John is taking a quiz that has three multiple-choice questions. Each question consists of four choices (A, B, C, and D) and has only one correct answer. Suppose that he did not study for the quiz and had to randomly guess the answer to each question; as such, each answer is selected independently of all other answers. What is the probability that he will have **only one** correct answer if he answers all three questions

A. $\frac{27}{64}$ B. $\frac{1}{27}$ C. $\frac{3}{16}$ D. $\frac{9}{64}$ E. $\frac{1}{4}$

27. Which option gives you a better chance to win \$5?

Option I. One ticket is drawn at random from Box A and one from Box B. Box A has three tickets with the numbers 4,9,9 written on them. Box B has four tickets marked 4,5,7,9. You will win \$5 if the number from Box A is the same as Box B. Otherwise, you win nothing and lose nothing.

Option II. A standard die is rolled twice. You will win \$5 if the total number of spots is at least 7, and the value of the second roll is at least as large as the first roll.

A. **Option I** gives the better chance of 1/4.

B. **Option II** gives the better chance of 1/3.

C. **Option I** gives the better chance of 1/6.

D. **Option II** gives the better chance of 7/12.

E. Both options give the same chance of 1/3.

28. A certain traffic light remains red for 50 seconds at a time. You arrive at random and find it red. What is the probability that you will have to wait more than 20 seconds for the light to turn green?

A. $\frac{1}{10}$ B. $\frac{2}{5}$ C. $\frac{4}{5}$ D. $\frac{3}{5}$ E. $\frac{1}{5}$

29. There are T trout and K catfish in a lake. Fish are caught randomly and eaten until only one type of fish remains. If one of the catfish is named Dan, what is the probability that Dan is not eaten?

A. $\frac{1}{K+1}$ B. $\frac{1}{T+K}$ C. $\frac{T}{T+K}$ D. $\frac{K}{T+K}$ E. $\frac{1}{T+1}$

30. The equation $2r = \cos\theta$ in polar coordinates represents a circle

A. centered at C($\frac{1}{4}$,0) of radius $\frac{1}{4}$

B. centered at C(0,$\frac{-1}{4}$) of radius $\frac{1}{4}$

C. centered at C($\frac{-1}{4}$,0) of radius $\frac{1}{2}$

D. centered at C(0, $\frac{1}{4}$) of radius $\frac{1}{4}$

E. centered at C($\frac{1}{4}$,0) of radius $\frac{1}{2}$