Utah State Mathematics Contest
Senior Exam
March 16, 2011

1. Making only vertical or horizontal moves between adjacent letters, in how many ways can you start from an *'M'* and move to the *'H'*, spelling the word *'MARCH'* ?

```
        M
      M A M
    M A R A M
  M A R C R A M
M A R C H C R A M
  M A R C R A M
    M A R A M
      M A M
        M
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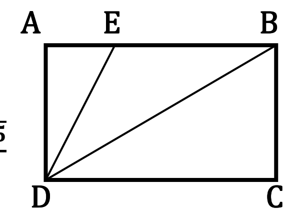
(a) 64 (b) 112 (c) 120 (d) 60 (e) 124

2. In rectangle ABCD, AD = 1, E is on AB, and DB and DE trisect $\angle ADC$. What is the perimeter of ΔBDE?



(a) $3+\frac{\sqrt{3}}{3}$ (b) $2+\frac{4\sqrt{3}}{3}$ (c) $2+2\sqrt{2}$ (d) $2+\frac{5\sqrt{3}}{3}$ (e) $\frac{3+\sqrt{5}}{2}$

3. If $A = Pe^{rt}$ (where A, P, r, and t are all positive), what is the value of t when A is five times as much as P ?

(a) $5r$ (b) $\frac{1}{5}r$ (c) $\ln(5r)$ (d) $\frac{\ln(r)}{5}$ (e) $\frac{\ln(5)}{r}$

4. If the price of movie tickets goes up by p percent, what percent of decrease in ticket sales will result in no net change in income?

(a) $\frac{1}{1-p}$ (b) $\frac{p}{1+p}$ (c) $\frac{1-p}{p^2}$ (d) $\frac{1}{1+p}$ (e) $\frac{p}{1-p}$

5. The Utah Jazz and the LA Lakers play a five-game playoff series – the first team to win three games wins the series. Each team is equally likely to win each game, there are no ties, and the outcomes of the individual games are independent. If the Lakers were to win Game 2, but the Jazz were to win the entire series, what is the probability that the Lakers won Game 1?

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$

6. For how many positive integers, n , less than or equal to 24, is $n!$ evenly divisible by the sum $1+2+\cdots+n$? ($n! = n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot 3\cdot 2\cdot 1$)

(a) 21 (b) 12 (c) 16 (d) 8 (e) 17

7. Let $a_1, a_2, \ldots$ be a sequence in which $a_1 = 7, a_2 = 10$, and $a_n = \frac{a_{n-1}}{a_{n-2}}$ for each integer $n \geq 3$. What is the value of a_{2011}?

(a) $\frac{10}{7}$ (b) 10 (c) $\frac{1}{7}$ (d) 7 (e) $\frac{7}{10}$

8. Two perpendicular lines both pass through the point $(6, 8)$. These two lines have y-intercepts that have a sum of zero. Determine the slope of the line with negative slope.

(a) $-\frac{1}{3}$ (b) $-\frac{1}{5}$ (c) $-\frac{1}{7}$ (d) $-\frac{1}{2}$ (e) $-\frac{3}{4}$

9. Consider the sets $\{0\}, \{1, 2\}, \{3, 4, 5\}, \{6, 7, 8, 9\} \ldots$ where the first set contains only the integer 0. For $n > 1$, the n^{th} set contains n consecutive integers, and the first element in each set is the next integer after the last element in the preceding set. If S_n is the sum of the elements in the n^{th} set, find S_{15}.

(a) 1120 (b) 570 (c) 1760 (d) 2240 (e) 1680

10. Thirty collectible action figures sell for an average of $20 each. Twenty more sell for an average of $30 each. What is the average price of all fifty collectibles?

(a) $23 (b) $24 (c) $25 (d) $26 (e) $27

11. The number of inches in the perimeter of an equilateral triangle equals the number of square inches in the area of a circle circumscribed about the triangle. What is the radius, in inches, of the circle?

(a) $\frac{3\sqrt{2}}{\pi}$ (b) $\frac{3\sqrt{3}}{\pi}$ (c) $\sqrt{3}$ (d) $\frac{6}{\pi}$ (e) $\sqrt{3}\pi$

12. Concrete sections on the new portions of I-15 are 80 feet long. As a car drives over the seams where sections meet, a brief thump is heard by the passengers. The speed of the car in miles per hour is approximately equal to the number of thumps heard in how many seconds?

(a) 70 (b) 45 (c) 55 (d) 85 (e) 40

13. The sum of the infinite series $\frac{4}{5}+\frac{5}{5^2}+\frac{4}{5^3}+\frac{5}{5^4}+\cdots$ is:

(a) $\frac{25}{16}$ (b) $\frac{15}{14}$ (c) $\frac{3}{2}$ (d) $\frac{5}{4}$ (e) $\frac{25}{24}$

14. Which if the following functions is defined as an odd function?

(a) $f(x) = \frac{x^3}{\sqrt{4x^2+1}}$ (b) $f(x) = x^3 + 5x + 9$ (c) $f(x) = \cos(7x)$
(d) $f(x) = \tan x + \sec x$ (e) $f(x) = 5x^2 + 3$

15. A traveler makes a 280 mile trip at an average speed of 7 miles per hour, but meanders home at an easy 4 miles per hour (yes, it's a long trip). The traveler's average speed for the entire trip is closest to:

(a) 5.0 mph (b) 5.7 mph (c) 5.3 mph (d) 5.5 mph (e) 5.1 mph

16. How many integer values of x can satisfy the following inequality?

$$\log(x-2) + \log(9-x) \leq 1$$

(a) 4 (b) 2 (c) 0 (d) 6 (e) infinitely many values

17. A barrel contains a selection of colored cubes, each of which is yellow, blue, or green. The number of green cubes is at least half of the number of blue cubes, and at most one third of the number of yellow cubes. The cubes which are blue or green number at least 34. The minimum number of yellow cubes is:

(a) 61 (b) 27 (c) 36 (d) 44 (e) 52

18. By adding the same particular constant to each of the numbers in the sequence $7, 15, 27$, a geometric progression is created. What is the common ratio between terms in the progression?

(a) $\frac{4}{3}$ (b) $\frac{8}{5}$ (c) $\frac{5}{3}$ (d) $\frac{3}{2}$ (e) 2

19. Let $M = i^n + i^{-n}$, where $i = \sqrt{-1}$, and n is any integer. How many different values can M possibly take?

(a) 2 (b) 3 (c) 4 (d) 6 (e) infinitely many

20. If the expressions $x + 3$ and $x - 3$ are reciprocals, what is the total of all possible values for x?

(a) $\mathbf{0}$ (b) $\sqrt{10}$ (c) $\sqrt{8}$ (d) 3 (e) $\sqrt{10} - \sqrt{8}$

21. Let $f(x) = \frac{x}{1-x}$, where $x \neq 1$. Under such conditions, $f^{-1}(x)$ can be re-written as:

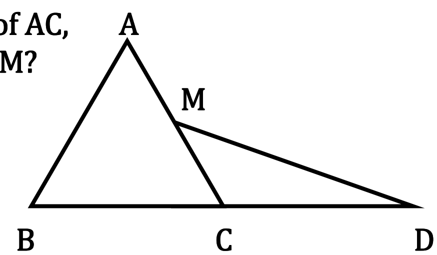
(a) $f\left(\frac{1}{x}\right)$ (b) $-f(x)$ (c) $-f(-x)$ (d) $f\left(-\frac{1}{x}\right)$ (e) $f(-x)$

22. There is a positive integer, B, such that $2B$ is a perfect square, $3B$ is a perfect cube, and $5B$ is a perfect fifth (an integer to the fifth power). Assume that B is the smallest such number. How many factors are in the prime factorization of B?

(a) 59 (b) 7 (c) 29 (d) 31 (e) 61

23. Equilateral ΔABC has side length 6. M is the midpoint of AC, and C is the midpoint of BD. What is the length of DM?

(a) $\frac{9\sqrt{2}}{2}$ (b) $3\sqrt{5}$ (c) $3\sqrt{6}$ (d) $3\sqrt{7}$ (e) $6\sqrt{2}$



24. Let x and y be two-digit integers such that y is obtained by reversing the digits of x. Suppose that the integers x and y also satisfy the equation $x^2 - y^2 = z^2$ for some positive integer z. What is the value of $x + y + z$?

(a) 144 (b) 116 (c) 88 (d) 154 (e) 112

25. If the radius of a circle is increased by 200%, then the area will be increased by:

(a) 200% (b) 800% (c) 300% (d) 900% (e) 400%

26. How many distinct real solutions are there to the following equation?

$$x^5 - x^4 + 7x^3 - 7x^2 + 12x - 12 = 0$$

(a) 5 (b) 4 (c) 3 (d) 2 (e) 1

27. What is the largest number of acute angles that can be found in a convex decagon?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

28. Assuming that c is a positive integer, what is the largest number which necessarily divides the expression $c^4 - c^2$:

(a) 24 (b) 3 (c) 6 (d) 2 (e) 12

29. A right circular cylinder with diameter equal to its height is inscribed in a right circular cone (the bases and axes of the two objects would coincide). The cone has diameter 10 and altitude 12. What is the radius of the cylinder?

(a) $\frac{8}{3}$ (b) $\frac{30}{11}$ (c) 3 (d) $\frac{25}{8}$ (e) $\frac{7}{2}$

30. Imagine two circles, the larger with center M and radius m, the smaller with center N and radius n. Constructing line segment MN, which of the following choices cannot be true of the length of the line segment connecting the two centers?

(a) It is equal to $m - n$ (b) It is less than $m - n$ (c) It is equal to $m + n$

(d) It is less than $m + n$ (e) Any of these may be true

31. If the graph of the equation $9(x - h)^2 + 4(y - k)^2 = 36$ is tangent to both the x- and y-axes, then the sum of h and k may be:

(a) 6 (b) 4 (c) -1 (d) 3 (e) -9

32. The areas (in square inches) of the front, top, and side of a rectangular box are 3, 10, and 12, respectively. Which of the following choices is the closest to the measure of the volume of the box in cubic inches?

(a) 22 (b) 27 (c) 19 (d) 16 (e) 25

33. In the equation $C = \frac{An}{B+nZ}$, let A, B, and Z be positive constants, and let n be a positive integer. As n increases, what happens to C?

(a) it increases (b) it decreases (c) it remains constant
(d) in increases, then decreases (e) it decreases, then increases

34. When the mean, median, and mode of the list $\{10, 2, 5, 2, 4, 2, x\}$ are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

(a) 6 (b) 20 (c) 9 (d) 3 (e) 17

35. The expression $\sqrt{5+2\sqrt{6}}-\sqrt{5-2\sqrt{6}}$ is equivalent to which of the following?

(a) $2\sqrt{6}$ (b) $2\sqrt{2}$ (c) $3\sqrt{5}$ (d) $3\sqrt{3}$ (e) $5\sqrt{2}$

36. Let m, n, and p be real numbers such that $8m+9n-p=12$ and $9m-8n+12p=1$. Find the sum of $m^2+n^2-p^2$.

(a) 12 (b) 0 (c) 8 (d) 1 (e) 9

37. If $\log_A B = \log_B A$, where $A > B > 0$, then the product AB is:

(a) 1 (b) 2 (c) 10 (d) $\frac{1}{2}$ (e) not a real number

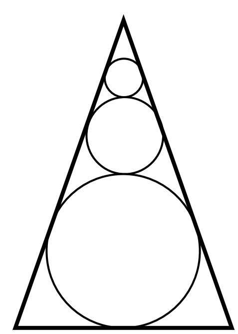
38. If $y = mx + b$ is the slant asymptote of $f(x) = \frac{5x^2-11}{x-3}$, then which is equal to $m^2 + b$?

(a) 10 (b) 29 (c) 40 (d) 59 (e) 74

39. Let P be the product of the numbers 3162011, 6102011, and 7122011. The value of $\log P$ is closest to which of the following?

(a) 19 (b) 22 (c) 20 (d) 23 (e) 21

40. Ben draws an image of Frosty the Snowman, comprised of three circles of radii 1, 2, and 4 inches. He then draws a triangular shade over Frosty (to keep out the sun) which runs tangent to Frosty's circular parts (which are, of course, tangent to each other). What is the area enclosed by the triangular shade?



(a) $\frac{128\sqrt{2}}{3}$ (b) $\frac{128\sqrt{3}}{3}$ (c) $32\sqrt{5}$ (d) $\frac{128\sqrt{3}}{3}$ (e) $64\sqrt{2}$

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1. (d) 60

Carving out one corner of the shape and re-orienting gives the following image. There are $2^4 = 16$ ways for the 'H' to trace out to the 'M'. Four times over would be 64, but that would be double-counting the far-left/right paths (the vertical/horizontal paths in the original image) each twice.

```
            H
         C     C
      R     R     R
   A     A     A     A
M     M     M     M     M
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2. (b) $2 + \frac{4\sqrt{3}}{3}$

The trisected right angle forms 30° angles, so two of the three triangles are 30-60-90 triangles. Length CD must be $\sqrt{3}$. Length BD must be 2. Length DE is $\frac{2\sqrt{3}}{3}$. ΔBED is isosceles, so length BE is also $\frac{2\sqrt{3}}{3}$.

3. (e) $\frac{\ln(5)}{r}$

Solving for t gives $= \frac{\ln\left(\frac{A}{P}\right)}{r}$. Since $A = 5P$, it must be (e).

4. (b) $\frac{p}{1+p}$

An increase in price by p percent is equivalent to multiplying by $(1 + p)$. This would be counterbalanced by multiplying sales by $\frac{1}{1+p}$. This represents a decrease of $\frac{p}{1+p}$.

5. (a) $\frac{1}{5}$

Assuming that each team is equally likely to win each game, and that each outcome is independent, then a five-game series, played to completion, has 32 equally likely possible outcomes. There are five which fit the given conditions. LLJJJ, JLLJJ, JLJLJ, JLJJL, JLJJJ. (Granted, the fourth and fifth scenarios cannot occur; however, that would make the 4-game outcome JLJJ twice as likely as any specific completed five-game scenario, as it encompasses the two outcomes JLJJL and JLJJJ) In only one of these five equally likely situations do the Lakers win Game 1.

6. (c) 16

The sum of the first n integers is equivalent to $\frac{n(n+1)}{2}$. For this to divide $n!$, $\frac{n!}{\frac{n(n+1)}{2}} = \frac{2\cdot(n-1)!}{(n+1)}$ must be an integer. For this to occur, $(n + 1)$ must be a composite number (or it must be 2). For $n \le 24$, n will be composite if it is a member of the following set: $\{1, 3, 5, 7, 8, 9, 11, 13, 14, 15, 17, 19, 20, 21, 23, 24\}$.

7. (d) 7

Writing out the sequence gives the following pattern: $\left\{7, 10, \frac{10}{7}, \frac{1}{7}, \frac{1}{10}, \frac{7}{10}, 7, 10, \frac{10}{7}, \frac{1}{7}, \frac{1}{10}, \frac{7}{10} \ldots\right\}$. The cycle repeats every six steps. Since 2011 is 1 (mod 6), then the 2011th step must be a 7.

8. (a) $-\frac{1}{3}$

The equations of the two lines are $y = ax + b$ and $y = -\frac{1}{a}x - b$. Using the given point, the equations become $8 = 6a + b$ and $8 = -\frac{6}{a} - b$. Solving this system of equations gives $a = 3$ and $b = -10$.

9. (e) 1680

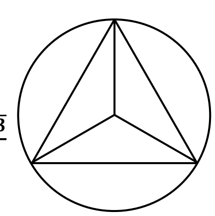
The first element in the n^{th} set is $\frac{n(n-1)}{2}$ (0, 1, 3, 6, 10... are triangular numbers). Thus, the first element in the 15th set is 105, and the last element would be 119. The sum of these fifteen consecutive integers is $\frac{119\cdot120}{2} - \frac{104\cdot105}{2}$ (the sum of the first 119 integers minus the sum of the first 104 integers), or 1680.

10. (b) \$24

Fifty figures sell for a total of \$1200. When combined, the average price should be \$1200/50=\$24.

11. (b) $\frac{3\sqrt{3}}{\pi}$

Let the perimeter of the triangle be $6x$, where each side length is $2x$. Then the height of the triangle, using 30-60-90 proportions, is $\sqrt{3}x$. As well, the center of the circle/triangle is two-thirds of the height of the triangle, making the radius $\frac{2\sqrt{3}}{3}x$, and the area $\frac{4}{3}\pi x^2$. Solving $6x = \frac{4}{3}\pi x^2$ gives $x = \frac{9}{2\pi}$, or a radius of $\frac{3\sqrt{3}}{\pi}$



12. (c) 55

$$\frac{x\ miles}{1\ hr}\cdot\frac{1hr}{3600\ sec}\cdot\frac{5280\ ft}{1\ mile}\cdot\frac{1\ thump}{80\ ft}=\frac{5280\cdot x\ thumps}{80\cdot 3600\ sec}=\frac{11x\ thumps}{600\ sec}\approx\frac{x\ thumps}{54.5\ sec}$$

13. (e) $\frac{25}{24}$

This is the sum of two geometric series. $\left(4\sum_{i=1}^{\infty}\frac{1}{5^i}\right)+\left(\sum_{i=1}^{\infty}\frac{1}{5^{2i}}\right)=\frac{\frac{4}{5}}{1-\frac{1}{5}}+\frac{\frac{1}{25}}{1-\frac{1}{25}}=1+\frac{1}{24}=\frac{25}{24}$.

14. (a) $f(x)=\frac{x^3}{\sqrt{4x^2+1}}$

Of the given options, only in the case of this function is it true for all x that $f(-x)=-f(x)$.

15. (e) 5.1 mph

The first trip takes 40 hours, while the return takes 70 hours. The combined distance of 560 miles divided by the combined time of 110 hours makes for an average speed of approximately 5.09 mph.

16. (a) 4

$\log(x-2)+\log(-x+9)=\log(-x^2+11x-18)\leq 1$. This is equivalent to the equation $(-x^2+11x-18)\leq 10$. Solving the quadratic inequality yields $\{x|x\leq 4\ or\ x\geq 7\}$. The domain of the original expression is $\{x|2<x<9\}$. The only integer values which can satisfy both parameters are the set $\{3,4,7,8\}$.

17. (c) 36

Let the number of yellow, green, and blue cubes be Y,G,B, respectively. Translating the given information (and multiplying out denominators) gives $2G\geq B$ and $3G\leq Y$. As well, $B+G\geq 34$. Substituting, $2G+G\geq B+G\geq 34$, or $3G\geq 34$. Since Y,G,B are positive integers, $G\geq 12$. Further, $Y\geq 3G\geq 36$.

18. (d) $\frac{3}{2}$

Using the given information, $\frac{7+x}{15+x}=\frac{15+x}{27+x}$. Solving gives $x=9$. The first three terms in the geometric sequence are 16, 24, 36, and the common ratio is $\frac{3}{2}$.

19. (b) 3

For integer values of n, the expression can take the values $\{1+1, i-i, -1-1, -i+i\}$, or $\{2,0,-2\}$.

20. (a) 0

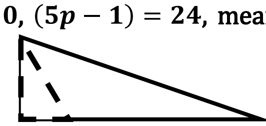
Given $x+3=\frac{1}{x-3}$, or $x^2-9=1$. Therefore, $x=\pm\sqrt{10}$, which add to a total of 0.

21. (c) $-f(-x)$

$f^{-1}(x)=\frac{x}{1+x}=-f(-x)$.

22. (a) 59

To minimize B, it must be assumed that the only possible prime factors are 2, 3 and 5. To meet the given conditions, $B=2^{2m-1}3^{3n-1}5^{5p-1}$, where m,n,p are all positive integers. Also, $(2m-1)$must be a multiple of 3 and 5, $(3n-1)$ must be a multiple of 2 and 5, and $(5p-1)$must be a multiple of 2 and 3. For the smallest possible outcome, $(2m-1)=15$, $(3n-1)=20$, $(5p-1)=24$, meaning $B=2^{15}3^{20}5^{24}$, which has 59 total prime factors (not all distinct).



23. (d) $3\sqrt{7}$

Drawing an altitude for △CDM, the small ghost shape is a 30-60-90 triangle. Given that the hypotenuse is 3, the height must be $\frac{3\sqrt{3}}{2}$ and the base must be $\frac{3}{2}$. As well, CD must have length 6. Now, the combo right triangle (pictured) has base $\frac{15}{2}$ and height $\frac{3\sqrt{3}}{2}$. By the Pythagorean Theorem, the hypotenuse is $\sqrt{\frac{225}{4}+\frac{27}{4}}=\sqrt{63}=3\sqrt{7}$.

24. (d) 154

Let $x=10a+b$ and $y=10b+a$, where a,b are one-digit positive integers. Then $x^2-y^2=(10a+b)^2-(10b+a)^2=99a^2-99b^2=99(a-b)(a+b)$. This can only be a square if 11 divides $(a-b)(a+b)$. Since these are one-digit numbers, $a+b=11$. As well, $a-b$ is a square, and in this case must be 1. Now, $a=6$, $b=5$, leading to $x=65$, $y=56$, and $z=\sqrt{99\cdot 11\cdot 1}=33$.

25. (b) 800%

Increasing the radius by 200% is equivalent to tripling the radius – not, as so many might instantly interpret, doubling it. Therefore, the area is nine times the original (or 800% bigger).

26. (e) 1

$x^5 - x^4 + 7x^3 - 7x^2 + 12x - 12 = (x-1)(x^4 + 7x^2 + 12) = (x-1)(x^2+3)(x^2+4)$, which has one real root and four non-real roots.

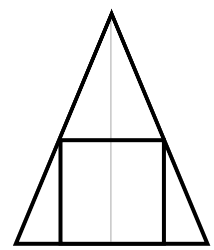
27. (b) 3

The sum of the exterior angles of any convex polygon is 360°. If any interior angle is acute, then the associated exterior angle must be obtuse, or greater than 90°. Since 4 or more obtuse angles would necessarily add to make more than 360°, there cannot be more than three acute interior angles. It is worth noting that the number of sides of the polygon is completely irrelevant to the problem.

28. (e) 12

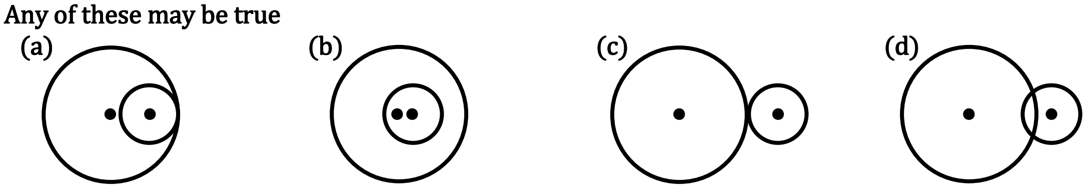
$c^4 - c^2 = (c-1)(c^2)(c+1)$ is the product of three consecutive non-negative integers (with the middle factor duplicated once). One of these three numbers must be a multiple of three. As well, if c is even, c^2 is a multiple of 4. If c is odd, both $(c-1)$ and $(c+1)$ are even. This expression must be a multiple of both 3 and 4, or a multiple of 12.

29. (b) $\frac{30}{11}$



Looking at the left side of a cross section of the inscribed shapes, there are two distinct similar right triangles above and to the left of the square (cylinder). If the left half of the square has base x and height $2x$, then the upper-left triangle has base x and height $12 - 2x$, and the lower-left triangle has base $5 - x$ and height $2x$. As these triangles are similar, the proportion $\frac{2x}{5-x} = \frac{12-2x}{x}$ must hold. Solving for x gives $\frac{30}{11}$, which is the radius of the cylinder.

30. (e) Any of these may be true



31. (c) -1

This is the equation of an ellipse with horizontal- and vertical-axis lengths of 2 and 3, respectively. Given that it is tangent to the x- and y-axes, then it must be centered at $(2,3)$, $(2,-3)$, $(-2,3)$, or $(-2,-3)$. Thus, the sum of h and k can only be 5, 1, -1, or 5.

32. (c) 19

Let the length, width, and height be called L, W, H, respectively. From the given information, $L \cdot W = 10$, $L \cdot H = 3$, $W \cdot H = 12$. It follows that $L \cdot W \cdot L \cdot H \cdot W \cdot H = L^2W^2H^2 = (LWH)^2 = 360$. Thus the volume is $\sqrt{360} \approx 19$.

33. (a) it increases

With respect to n, C is a ratio of two linear expressions. Assuming all other variables are constant, $\lim_{n\to\infty} C = \frac{A}{Z} > 0$. If $n = 0$, then $C = 0$. So, as $n \to \infty$, C must be strictly increasing from $0 \to \frac{A}{Z}$.

34. (b) 20

The mode is 2, the mean is $\frac{25+x}{7}$, and the median is 2, 4, or x. Since the numbers form a non-constant arithmetic progression, the median is not 2. For the median to be 4, $x \geq 4$. If the median is 4, the mean must be 0, 3 or 6 (to maintain an arithmetic progression). By this reasoning, $x = -25$, $x = -4$ or $x = 17$. However, $x = -4$ and $x = -25$ create contradictions. For the median to be x, it must be the case that $2 < x < 4$, giving a unique solution of $x = 3$. So, x is 3 or 17.

35. (b) $2\sqrt{2}$

Call $A = \sqrt{5 + 2\sqrt{6}}$ and $B = \sqrt{5 - 2\sqrt{6}}$. It follows that $A^2 = 5 + 2\sqrt{6}$, $B^2 = 5 - 2\sqrt{6}$, and $AB = \sqrt{25 - 24} = 1$. $(A-B)^2 = A^2 - 2AB + B^2 = 5 + 2\sqrt{6} - 2 + 5 - 2\sqrt{6} = 8$. Therefore, $A - B = \sqrt{8} = 2\sqrt{2}$.

36. (d) 1

Rearrange the equations to create $8m + 9y = p + 12$ and $9m - 8y = -12p + 1$. Squaring both yields $64m^2 + 144mn + 81n^2 = p^2 + 24p + 144$ and $81m^2 - 144mn + 64n^2 = 144p^2 - 24p + 1$. Add the square equations to give $145m^2 + 145n^2 = 145p^2 + 145$. Dividing both sides by 145 gives $m^2 + n^2 = p^2 + 1$. Lastly, $m^2 + n^2 - p^2 = 1$.

37. (a) 1

Using the change-of-base formula, $\frac{\log B}{\log A} = \frac{\log A}{\log B}$, or $(\log B)^2 = (\log A)^2$. Now, $|\log B| = |\log A|$. Since $A \neq B$, then $A = \frac{1}{B}$, meaning that $AB = 1$.

38. (c) 40

Dividing the fraction yields $5x + 15 + \frac{34}{x-3}$. The slant asymptote is $5x + 15$. Thus, $m^2 + b = 40$.

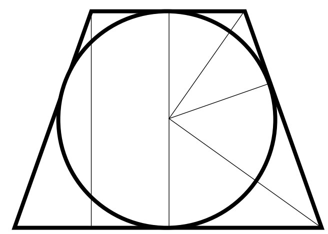
39. (c) 20

Call the three numbers a, b, c, where $P = abc$. Then:

$3 \cdot 10^6 < a < 4 \cdot 10^6$ $\quad$ $6 \cdot 10^6 < b < 7 \cdot 10^6$ $\quad$ $7 \cdot 10^6 < c < 8 \cdot 10^6$

This leads to the fact that $126 \cdot 10^{18} < P < 224 \cdot 10^{18}$, or $1.26 \cdot 10^{20} < P < 2.24 \cdot 10^{20}$. Since $\sqrt{10} > 3$, then $20 < \log P < 20.5$.

40. (e) $64\sqrt{2}$



The height of the triangular shade can be thought of as a decreasing geometric series of similar trapezoids, where the first ball has diameter 8, then 4, then 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, etc., and the height of the triangle is $\frac{8}{1-\frac{1}{2}} = 16$. Each ball can be considered a similar part of a geometric series of shapes. Consider the lowest one on the original diagram. The height of the attached figure is 8, and the right side of the shape is dissected into two pairs of congruent triangles (proven through side-side-side correspondence). If the lower base of this trapezoid is $4x$, and the upper base is $2x$, then the slant side of the trapezoid must be $2x + x = 3x$ (through the congruent triangles). Looking at the left-most triangle in the diagram, the sides are $x, 8, 3x$. By the Pythagorean Theorem, $x^2 + 8^2 = (3x)^2$, which solves to $x = 2\sqrt{2}$. Thus, the base of the trapezoid is $8\sqrt{2}$. The base and height of the original triangle are 16 and $8\sqrt{2}$, giving an area of $64\sqrt{2}$.