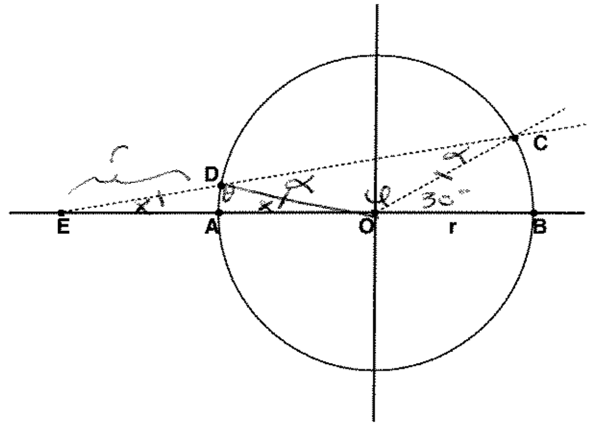
State Senior Mathematics Contest
Spring 2008

1. Points A, B, C lie on a circle with radius r centered at O. Segment DE has length r.



If $m(< BOC) = 30°$, then $M(< BEC) =$

(*a*) 15°
(*b*) 12°
(*c*) 10°
(*d*) 20°
(*e*) cannot be determined without more infomation

$x = ?$

ΔEDO is isosceles $\Rightarrow$ base angles x are same

$2x+\theta=180°$ but also $\theta+\alpha=180°$

$\Rightarrow \alpha = 2x$

ΔDOC is isosceles $\Rightarrow \angle ODC \cong \angle OCD$

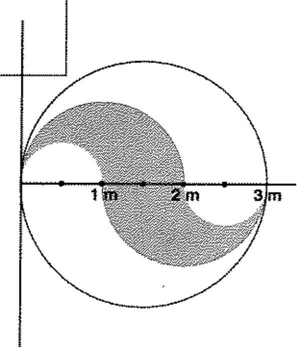
$\varphi + 2\alpha = 180°$ and $x + \varphi + 30° = 180°$

$\varphi = 180° - 2\alpha$ $\Rightarrow$ $x + 180° - 4x + 30° = 180°$

$\varphi = 180° - 4x$ $\quad -3x = -30°$

$x = 10°$

2. The circle to the right has diameter 3 m. Find the area of the shaded region if its boundary consists of semicircles:



(*a*) $3\pi\, m^2$ (*b*) $\pi\, m^2$ (*c*) $\frac{3}{2}\pi\, m^2$ (*d*) $\frac{3}{4}\pi\, m^2$
(*e*) $\frac{3}{8}\pi\, m^2$

$$A_{shaded} = 2\left(\frac{\pi(1^2)}{2} - \frac{\pi(\frac{1}{2})^2}{2}\right)$$

$$= \pi - \frac{1}{4}\pi = \frac{3}{4}\pi \quad m^2$$

3. If f is a function such that $f(x-1) = x^2 - 3x + 5$ then $f(x+1) = ?$

(a) $x^2 + x + 3$
(b) $x^2 - x + 3$
(c) $x^2 + x$
(d) $x^2 - 3x + 7$
(e) none of these

We know $f(x-1) = x^2 - 3x + 5$

Let $y = x - 1 \Leftrightarrow x = y + 1$.

Then $f(y) = f(x-1) = x^2 - 3x + 5$

$f(y) = (y+1)^2 - 3(y+1) + 5$

$f(y) = y^2 + 2y + 1 - 3y - 3 + 5$

$f(y) = y^2 - y + 3$

or $f(x) = x^2 - x + 3$

$\Rightarrow f(x+1) = (x+1)^2 - (x+1) + 3$

$= x^2 + 2x + 1 - x - 1 + 3$

$= x^2 + x + 3$

4. Given that $\begin{bmatrix} a & -2 \\ 1 & d \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a possible solution for d.

(a) $\sqrt{2}$ (b) -1 (c) 1 (d) $\sqrt{3}$ (e) none of these

$$\begin{bmatrix} a & -2 \\ 1 & d \end{bmatrix}^2 = \begin{bmatrix} a & -2 \\ 1 & d \end{bmatrix}\begin{bmatrix} a & -2 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a^2-2 & -2a-2d \\ a+d & -2+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow a^2-2=1$ $\quad -2a-2d=0 \quad a+d=0$

$a^2=3 \quad -2a=2d \quad a=-d$

$a=\pm\sqrt{3} \quad a=-d$

$-2+d^2=1$

$d^2=3$

$d=\pm\sqrt{3}$

5. Given a collection of three numbers, the smallest is zero. If the mean of the three numbers is $\log 4$ and the median is $\log 5$, then the largest is:

(a) $\log 6$ (b) $\log 9$ (c) $\log 10.4$ (d) $\log 12.8$ (e) $\log 20$

x, y, z = three numbers

Let x be smallest, i.e. $x=0$

$\frac{x+y+z}{3} = \log 4$

Let $x<y<z$. Then median $= y = \log 5$

$\Rightarrow \frac{0+\log 5+z}{3} = \log 4$

$\log 5 + z = 3\log 4$

$z = 3\log 4 - \log 5$

$z = \log 4^3 - \log 5 = \log\left(\frac{4^3}{5}\right) = \log(12.8)$

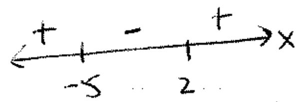
6. What is the probability that a solution for $x^2+3x<10$ is also a solution for $x^2>5$?

(a) $\frac{2+\sqrt{5}}{14}$ (b) $\frac{\sqrt{5}}{7}$ (c) $\frac{5-\sqrt{5}}{7}$ (d) $\frac{2\sqrt{5}}{7}$
(e) none of these

$x^2+3x<10$

$x^2+3x-10<0$

$(x+5)(x-2)<0$



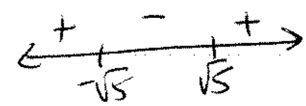
Solution: $x\in(-5,2)$

or $-5<x<2$

$x^2>5$

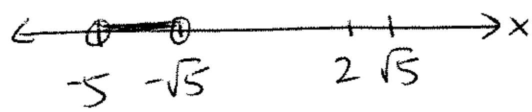
$x^2-5>0$

$(x-\sqrt{5})(x+\sqrt{5})>0$



solution:

$x\in(-\infty,-\sqrt{5})\cup(\sqrt{5},\infty)$

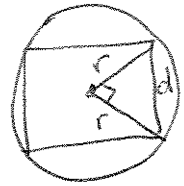


$$\frac{P(x>\sqrt{5} \text{ or } x<-\sqrt{5})}{P(-5<x<2)}=\frac{-\sqrt{5}-(-5)}{2-(-5)}$$

$$=\frac{-\sqrt{5}+5}{7}=\frac{5-\sqrt{5}}{7}$$

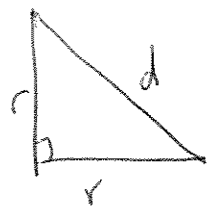
7. The ratio of the circumference of a circle to the perimeter of an inscribed square is:

(a) $\frac{\pi\sqrt{2}}{3}$ (b) $\frac{\pi\sqrt{2}}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$ (e) none of these



$$\frac{C_\circ}{P_\square} = \frac{2\pi r}{4\sqrt{2}\, r} = \frac{\pi}{2\sqrt{2}}$$

$$= \frac{\pi}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}\,\pi}{4}$$



$$r^2 + r^2 = d^2$$
$$2r^2 = d^2$$
$$\sqrt{2}\, r = d$$
$$\Rightarrow P_\square = 4d = 4(\sqrt{2}\, r) = 4\sqrt{2}\, r$$

8. Which of the following conditions imply that the real number x is rational?

I. $\sqrt{x}$ is rational
II. x^2 and x^3 are rational
III. x^2 and x^4 are rational

(*a*) I only
(*b*) I and II only
(*c*) I and III only
(*d*) II and III only
(*e*) I, II, and III

(i) If $\sqrt{x}$ rational, then $\sqrt{x} = \frac{p}{q} \Rightarrow x = \frac{p^2}{q^2}$ which is still rational, $p, q \in \mathbb{Z}$, $q \neq 0$.
(ii) If x^2 and x^3 are rational, then $x^2 = \frac{p}{q}$, $p, q \in \mathbb{Z}$, $q \neq 0$. $\Rightarrow x = \pm\sqrt{\frac{p}{q}} \Rightarrow x^3 = \frac{\pm p\sqrt{p}}{q\sqrt{q}}$ since x^3 rational, then $\frac{\sqrt{p}}{\sqrt{q}}$ also rational, i.e. x rational.
(iii) Let $x = \sqrt{2}$, $x^2 = 2$, $x^4 = 4$, but x not rational.

9. The graph of $9x^2 - 54x + 4y^2 + 16y + 61 = 0$ is

- (a) a circle with a radius 6, centered at (2,-3)
- (b) an ellipse centered at (3,-2)
- (c) a hyperbola centered at (3, -2)
- (d) an ellipse centered at (2,-3)
- (e) a hyperbola centered at (2,-3)

$$9x^2 - 54x + 4y^2 + 16y + 61 = 0$$
$$9(x^2 - 6x) + 4(y^2 + 4y) = -61$$
$$9(x^2 - 6x + 9) + 4(y^2 + 4y + 4) = -61 + 81 + 16$$
$$9(x-3)^2 + 4(y+2)^2 = 36$$
$$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{9} = 1$$
ellipse centered at $(3, -2)$

10. How many six-digit numbers can be formed using the digits 1, 2, 3, 4, 5 and 6 that have at least two of the digits the same?

(a) 6! (b) 6^6 (c) $6(6^5 - 5!)$ (d) 6^5 (e) $6^6 - 5!$

#s w/ at least 2 digits same

= total # of #s − #s w/ no digits same

$= 6^6 - 6! = 6(6^5 - 5!)$

11. Ten students solved a total of 35 problems in a contest (each problem solved by only one student). At least one student solved only one problem, at least one student solved exactly two problems, and at least one student solved exactly three problems. At least how many problems did the student who solved the most problems solve?

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6 (*e*) 7

10 students

35 problems

x students solved only one problem $x \geq 1$

y " " exactly 2 problems $y \geq 1$

z " " " 3 " $z \geq 1$

There are $35-(1+2+3) = 29$ problems left (at most), among 7 students.

If 6 of these students solved only 1, then last student solved 23. But we want the least # of problems the best student solved.

$29 \div 7 = 4 \text{ r } 1$ ⇒ i.e. 6 students solved 4 problems and last student solved 5

12. Add all the digits in 2008^{2008}. Add all the digits in the resulting number. Keep going until the result has only one digit. What is this digit?

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

You can recognize a pattern.

n	2008^n	sum of digits
0	1	1
1	2008	1
2	4032064	1
3	8,096,384,512	1
4	16,257,549,100,096	1

OR know that sum of digits of $2008^{2008} = (2008 \bmod 9)^{2008}$

$= 1^{2008} = 1$

13. On a trip, you run at a pace of 6 minutes per mile for a certain time and then at a pace of 12 minutes per mile for the same amount of time. What is your average pace for the entire trip, in minutes per mile?

(a) 7 (b) 8 (c) 9 (d) 10 (e) 11

$6 \frac{\text{min}}{\text{mile}} \to d_1 = \frac{1 \text{ mi}}{6 \text{ min}} t$ (you need harmonic avg here)

$12 \frac{\text{min}}{\text{mile}} \to d_2 = \frac{1 \text{ mi}}{12 \text{ min}} t$

$$V_{avg} = \frac{\text{total dist}}{\text{total time}} = \frac{t/6 + t/12}{2t}$$

$$= (1/6 + 1/12)\frac{1}{2} = \frac{3}{12}\left(\frac{1}{2}\right) = \frac{1}{8} \frac{\text{mile}}{\text{min}}$$

or $\frac{8 \text{ min}}{1 \text{ mile}}$

14. Find all the solutions on $[0, 2\pi)$.

$$\sin(4\theta) - \cos(2\theta) = 0$$

(a) $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

(b) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

(c) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$

(d) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

(e) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

$\sin(4\theta) - \cos(2\theta) = 0$

$2\sin(2\theta)\cos(2\theta) - \cos(2\theta) = 0$

$\cos(2\theta)\left(2\sin(2\theta) - 1\right) = 0$

$\cos(2\theta) = 0$ or $2\sin(2\theta) - 1 = 0$

$2\theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$

$\sin(2\theta) = 1/2$

$2\theta = \pi/6, 5\pi/6, 13\pi/6, 17\pi/6$

$\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

$\theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$

15. How many three-digit whole numbers have the property that doubling them results in reversing their digits? (For instance 125 does not have this property since 2(125) = 250 which does not equal 521, the number obtained by reversing the digits of 125. Also, 025 is not considered a three-digit number, but rather the two-digit number 25.)

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 6 (*e*) none of the above

let original # be abc w/ value
$100a+10b+c$.
Then we need $200a+20b+2c$
$= 100c+10b+a$

leaves us 4 cases
① $2a=c$, $2b=b$, $2c=a$ ⟹ $a=b=c=0$
② $2a=c$, $b=2b+1$, $a=2c-10$
⟹ $b=-1/2$
③ $c=2a+1$, $b=2b-10$, $a=2c$
⟹ $c=-1/3$
④ $c=2a+1$, $b=2b-10+1$, $a=2c-10$
⟹ $c=19/4$
⟹ ∄ no positive integer solution.

16. Mary had a coin purse with fifty coins (which are either pennies, nickles, dimes or quarters) totaling exactly \$1.00. Unfortunately, while counting her change, she dropped one coin. What is the probability that it was a penny?

(*a*) 50%
(*b*) 75%
(*c*) 85%
(*d*) 90%
(*e*) There is not enough information to determine the answer.

$0.01p + 0.05n + 0.1d + 0.25q = 1$

$\Leftrightarrow \quad p + 5n + 10d + 25q = 100 \quad ①$

$p + n + d + q = 50 \quad ②$

$① - ② \Rightarrow 4n + 9d + 24q = 50$

$\Leftrightarrow \quad n = \dfrac{50 - 9d - 24q}{4}$

From this, we can gather exhaustive list of choices.

	p	n	d	q
$\boxed{A}$	40	8	2	0
$\boxed{B}$	45	2	2	1

$P(\text{drop penny in } \boxed{A}) = \dfrac{40}{50} = 4/5$

$P(\text{drop penny in } \boxed{B}) = \dfrac{45}{50} = 9/10$

Since both $\boxed{A}$ + $\boxed{B}$ are equally likely, then $P(\text{drop penny}) = \dfrac{\left(\frac{4}{5} + \frac{9}{10}\right)}{2} = 85\%$

17. If $\log_8(\log_4(\log_2(x))) = 0$, express $x^{-2/3}$ as a real number

(a) 64 (b) 1/64 (c) $4\sqrt[3]{4}$ (d) $\sqrt[3]{2}/8$ (e) $-1/64$

$\log_8(\log_4(\log_2(x))) = 0$

$\Leftrightarrow \quad 8^0 = \log_4(\log_2 x)$

$1 = \log_4(\log_2 x)$

$\Leftrightarrow \quad 4^1 = \log_2 x$

$\Leftrightarrow \quad 2^4 = x \quad \Leftrightarrow \quad x = 16$

$\Rightarrow \quad x^{-2/3} = 16^{-2/3} = (2^4)^{-2/3} = 2^{-8/3} = \dfrac{1}{2^{8/3}}$

$= \dfrac{1}{4\sqrt[3]{4}} = \dfrac{\sqrt[3]{2}}{8}$

18. Given | S | T | A | T | E | | M | A | T | H | how many arrangements are there of these blocks?

(a) 10! (b) $\frac{10!}{5!}$ (c) $\frac{10!}{3!}$ (d) $\frac{10!}{12}$ (e) $\frac{10!}{6}$

If all the blocks were different, it would be 10!. But since some are the same, it's

$$\frac{10!}{3!\ 2!} = \frac{10!}{(3\cdot 2)(2)} = \frac{10!}{12}$$

because 3 $\boxed{T}$ and we can't tell difference

because 2 $\boxed{A}$ and we can't tell difference

19. Amaliea is putting her stack of pennies into rolls, keeping out the shiny ones. She notices that every other penny she picks up is dull and every third one is discolored and every fourth one is nicked or bent. How many pennies will she have to roll up if she ends up with fifty shiny pennies?

(a) 50 (b) 100 (c) 120 (d) 150 (e) 160

x = every other one dull d = every 3rd one discolored

$$\underbrace{\_\ x\ d\ x\ \_\ xd}\ \underbrace{\_\ x\ d\ x\ \_\ xd}$$

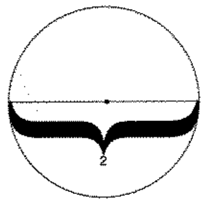
$$\underbrace{\_\ x\ d\ x\ \_\ xd}\ \_\ x\ d$$

The last fact that every 4th penny is nicked doesn't rule out any more pennies.

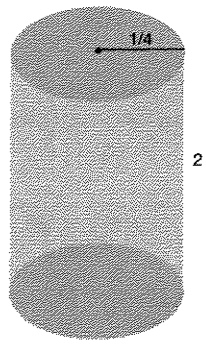
⇒ Basically, for every 6 pennies, we only keep out 2 shiny pennies and we roll up 4 pennies.

⇒ for 50 shiny pennies we have 25 groups of 6 total ⇒ $25(4) = 100$ rolled up

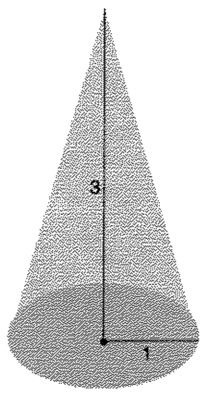
20. Which of the following is **not** equal to π?

(*a*) 

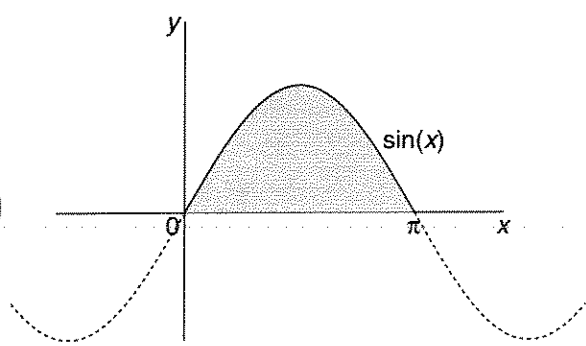
area of this circle

(*b*) 

surface area of the label for this cylinder

(*c*) 

volume of this cone

(*d*) 

shaded area under this curve

(*e*) all of the above are numerically equal to π

(a) $A = \pi(1^2) = \pi$

(b) $SA = 2\pi r h = 2\pi(\frac{1}{4})(2) = \pi$

(c) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(1^2)(3) = \pi$

(d) $A = \int_0^\pi \sin x \, dx = -\cos x \Big|_0^\pi$

$= -(-1-1) = 2 \neq \pi$

21. Ada speaks the truth and nothing but the truth every other day. On the other days she always lies. Today she made exactly four of the following five statements. Which statement did she not make today?

(a) I have a prime number of friends.
(b) Half of my friends are male.
(c) 288 is divisible by 12.
(d) I always speak the truth.
(e) Three of my friends are older than I.

We know that 288 is indeed divisible by 12. So (c) is true.
Assume (for a moment) she is telling truth today. Then (a) and (b) imply that Ada has 2 friends, since 2 is the only even prime #. But (e) contradicts that. And (a) w/ (e) ⇒ (b) is contradiction. That is, we can't determine which is false statement.
However, if four statements are false, then the only certain true statement is (c). And there are no contradictions w/ this assumption.

22. The Greek Alphabet has 24 letters. You want to name your fraternity with a 3-letter greek monogram, with no letter being used more than once. How many such monograms are possible? (Ignore the fact that some have been used by other fraternities.)

(a) $24!$ (b) $\frac{24!}{3!}$ (c) $\frac{24!}{21!}$ (d) $\binom{24}{3}$ (e) 24^3

$${}_{24}P_3 = \frac{24!}{21!} = 24(23)(22)$$

23. Which of the following statement is the incorrect one?

- (a) Between any two distinct real numbers there are infinitely many other real numbers.
- (b) Between any two distinct real numbers there are as many real numbers as between any other two distinct real numbers.
- (c) Between any two distinct real numbers there are as many real numbers as there are real numbers altogether.
- (d) Between any two distinct integers there are as many rational numbers as there are integers altogether.
- (e) Between any two distinct integers there are as many rational numbers as there are real numbers altogether.

This just requires an understanding of the difference between countably infinite (e.g. # of integers or rational #s) and uncountably infinite (e.g. # of real #s).

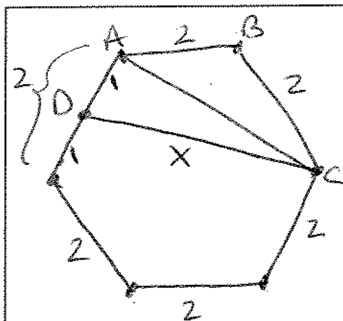
24. Suppose you want to tessellate the plane. Which of the following can you *not* use for this purpose?

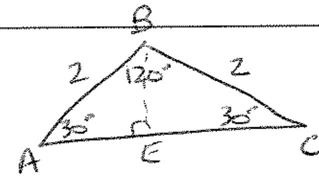
- (a) Regular Hexagons
- (b) Regular Pentagons
- (c) Trapezoids
- (d) Squares
- (e) Equilateral Triangles

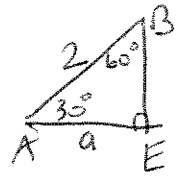
<u>All</u> triangles and quadrilaterals tesselate the plane and regular hexagons do as well.
That leaves only the pentagons from this list that don't tesselate.

25. A park has the shape of a regular hexagon of sides 2 km each. Allice walks a distance of 5km around the perimeter. What is the direct distance between the start point and the end point?

(a) $\sqrt{13}$ (b) $\sqrt{14}$ (c) $\sqrt{15}$ (d) $\sqrt{16}$ (e) $\sqrt{17}$







We know vertex angle for regular hexagon

$=\frac{180^\circ(6-2)}{6}$

$= 30(4)$

$= 120^\circ$

$m\angle BAC = 30^\circ$

$m\angle BAC + m\angle CAD = 120^\circ$

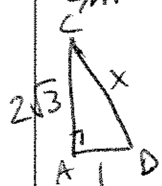
$\Rightarrow m\angle CAD = 90^\circ$

$\cos 30^\circ = \frac{a}{2}$

$\Rightarrow a = 2\cos 30^\circ$

$a = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

$\Rightarrow AC = 2\sqrt{3}$

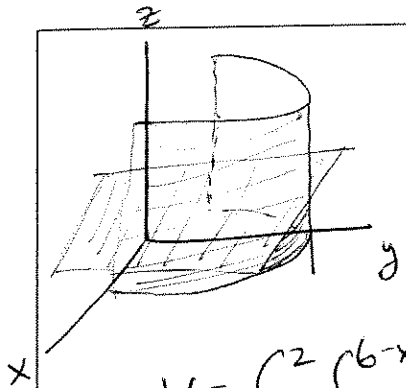


$1^2 + (2\sqrt{3})^2 = x^2$

$1 + 12 = x^2 \Rightarrow x = \sqrt{13}$

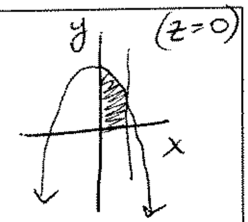
26. What is the volume of the solid in the first octant bounded by the plane $x + z = 2$ and the cylinder $y = 6 - x^2$.

(a) $\frac{32}{3}$ (b) $8\sqrt{6} - 9$ (c) $\frac{8}{3}$ (d) $\frac{32}{5}$ (e) $\frac{8}{5}$



$x+z=2$

$y=6-x^2$



$y=6-x^2$

$x=2$

$$V=\int_0^2\int_0^{6-x^2}\int_0^{2-x} dz\,dy\,dx$$

$$=\int_0^2\int_0^{6-x^2}(2-x)\,dy\,dx$$

$$=\int_0^2 (2-x)y\Big|_0^{6-x^2}dx$$

$$=\int_0^2 (2-x)(6-x^2-0)\,dx$$

$$=\int_0^2 12-2x^2-6x+x^3\,dx$$

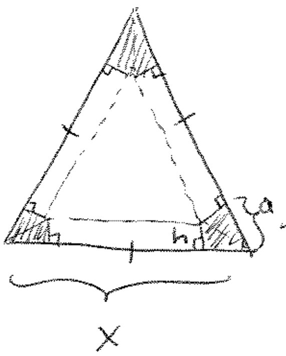
$$=\left(12x-\frac{2x^3}{3}-\frac{6x^2}{2}+\frac{x^4}{4}\right)\Big|_0^2$$

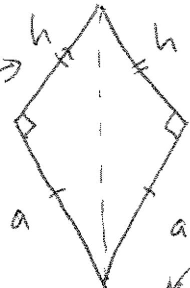
$$=\left(24-\frac{16}{3}-12+4\right)-0$$

$$=16-\frac{16}{3}=32/3$$

27. Given an equilateral triangular piece of cardboard, create an open box (i.e., without a lid) by cutting the same shape from each corner and folding up the flaps. What is the height of the box of maximal volume? (Assume length of the leg of original cardboard piece is x.)

(a) $\frac{x}{6}$ (b) $\frac{\sqrt{3}x}{18}$ (c) $\frac{x}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{9}x$ (e) $\frac{1}{3}x$





X is constant

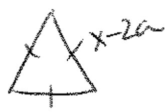
h = height of box

$a = h\cot 30^\circ \Rightarrow a = \sqrt{3}h$

maximize volume

$$V = Ah = \frac{\sqrt{3}}{4}(x-2\sqrt{3}h)^2 h$$

A = area of base



Area of equilateral Δ

$$= \frac{1}{2}(x-2a)\left(\frac{\sqrt{3}}{2}\right)(x-2a)$$

$$= \frac{\sqrt{3}}{4}(x-2a)^2$$

$$V = \frac{\sqrt{3}}{4}(x^2 - 4\sqrt{3}xh + 12h^2)h$$

$$= \frac{\sqrt{3}}{4}(x^2h - 4\sqrt{3}xh^2 + 12h^3)$$

$$\frac{dV}{dh} = \frac{\sqrt{3}}{4}(x^2 - 8\sqrt{3}xh + 36h^2) = 0$$

$$36h^2 - 8\sqrt{3}xh + x^2 = 0$$

$$h = \frac{8\sqrt{3}x \pm \sqrt{192x^2 - 4(36)(x^2)}}{2(36)}$$

$$h = \frac{8\sqrt{3}x \pm \sqrt{48x^2}}{72} = \frac{8\sqrt{3}x \pm 4\sqrt{3}x}{72}$$

$$h = \frac{12\sqrt{3}x}{72} \text{ or } \frac{4\sqrt{3}x}{72}$$

$$h = \frac{\sqrt{3}x}{6} \text{ or } \frac{\sqrt{3}x}{18}$$

and $V\left(\frac{\sqrt{3}x}{6}\right) = 0$

but $V\left(\frac{\sqrt{3}x}{18}\right) = x^2/54$ max volume

28. Find the convergence set for

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n+1}$$

(a) $2 < x < 4$
(b) $2 < x \le 4$
(c) $1 < x < 4$
(d) $1 < x < 5$
(e) $1 < x \le 5$

ART $\left|\frac{(x-3)^{n+1}}{2^{n+1}+1}\cdot\frac{2^n+1}{(x-3)^n}\right| = |x-3|\left|\frac{2^n+1}{2^{n+1}+1}\right|$

and

$\lim_{n\to\infty}|x-3|\left(\frac{2^n+1}{2^{n+1}+1}\right) \overset{L}{=} |x-3|\lim_{n\to\infty}\left(\frac{\ln 2}{\ln 2}\right)\left(\frac{2^n}{2^{n+1}}\right)$

$= \frac{|x-3|}{2}$

for convergence $\frac{|x-3|}{2} < 1 \iff |x-3| < 2$

$-2 < x-3 < 2$

$1 < x < 5$

check endpts

① $x=1$: $\sum_{n=0}^{\infty}\frac{(-2)^n}{2^n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n}{2^n+1}$

by AST $\Rightarrow$ diverges

② $x=5$: $\sum_{n=0}^{\infty}\frac{2^n}{2^n+1}$ $\quad \lim_{n\to\infty}\frac{2^n}{2^n+1} = 1$

by n^{th} term test, diverges

convergence set $\Rightarrow$ $1 < x < 5$

29. Given

$$f(x) = \begin{cases} \sqrt{10-x^2} & -3 < x < 3 \\ -e^{x-3}+2 & x \geq 3 \end{cases}$$

The graph of $f(x)$ is:

(*a*) continuous and differentiable at $x = 3$;
(*b*) continuous but not differentiable at $x = 3$;
(*c*) differentiable but not continuous at $x = 3$;
(*d*) neither continuous nor differentiable at $x = 3$;
(*e*) continuity and differentiability cannot be determined at $x = 3$.

$$f(x) = \begin{cases} \sqrt{10-x^2} & -3 < x < 3 \\ -e^{x-3}+2 & x \geq 3 \end{cases}$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \sqrt{10-x^2} = \sqrt{10-9} = 1$$

$$f(3) = -e^0 + 2 = -1+2 = 1$$

$\Rightarrow f(x)$ continuous at $x=3$

$$f'(x) = \begin{cases} \dfrac{-2x}{2\sqrt{10-x^2}} = \dfrac{-x}{\sqrt{10-x^2}} & -3 < x < 3 \\ -e^{x-3} & x \geq 3 \end{cases}$$

$$\lim_{x \to 3^-} f'(x) = \lim_{x \to 3^-} \frac{-x}{\sqrt{10-x^2}} = \frac{-3}{1} = -3$$

$$f'(3) = -e^0 = -1$$

not differentiable at $x=3$

30. Suppose three Utah polling organizations are in talks to work on a voter census. The first crew can complete the job in 36 days. The second crew can complete the job in 12 days. If all three crews work the job, they can complete it in 6 days. How long would it take for the third crew to do the job alone?

(*a*) 4 days (*b*) 8 days (*c*) 16 days (*d*) 18 days (*e*) 24 days

$$\frac{1}{36} + \frac{1}{12} + \frac{1}{x} = \frac{1}{6}$$

x = time for 3rd crew to do job

$$36x\left(\frac{1}{36} + \frac{1}{12} + \frac{1}{x}\right) = \left(\frac{1}{6}\right)36x$$

$$x + 3x + 36 = 6x$$

$$36 = 2x$$

$$18 = x$$