Problems for Spring 2026

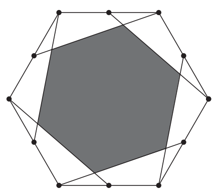
PROBLEMS

Proposals

To be considered for publication, solutions should be received by March 1, 2026.

2226. *Proposed by Robert Haas, Cleveland Heights, OH.*

Given a regular n-gon, form a smaller polygon by joining the midpoint of each side to the (clockwise) next nonadjacent vertex. (See the figure for an example when $n = 6$.) Find the ratio of the area of the smaller (shaded) polygon to that of the original polygon.



2227. *Proposed by Aleksandar Ilic, Menlo Park, CA.*

Given positive real numbers $x_1, x_2, \ldots, x_n$, let

$$M_k = \sum_{i=1}^{n} x_i^k.$$

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We invite readers to submit original problems appealing to students and teachers of advanced undergraduate mathematics. Proposals must always be accompanied by a solution and any relevant bibliographical information that will assist the editors and referees. A problem submitted as a Quickie should have an unexpected, succinct solution. Submitted problems should not be under consideration for publication elsewhere.

Proposals and solutions should be written in a style appropriate for this Magazine.

Authors of proposals and solutions should send their contributions using the Magazine*'s submissions system hosted at* http://mathematicsmagazine.submittable.com*. More detailed instructions are available there. We encourage submissions in PDF format, ideally accompanied by LaTeX source. General inquiries to the editors should be sent to* mathmagproblems@maa.org.

Find the least upper bound and greatest lower bound of

$$F(x_1, \ldots, x_n) = \frac{M_1^3 M_3}{M_2^3}.$$

2228. *Proposed by Hideyuki Ohtsuka, Saitama, Japan.*

Given $a > 0$ and $b > 0$. Let

$$R_n(a, b) = \sqrt{a + b\sqrt{a + b\sqrt{a + \cdots + b\sqrt{a}}}} \text{ (n square roots)}.$$

Find

$$\lim_{n\to\infty} \sum_{k=1}^{n} \prod_{j=k}^{n} \frac{b}{R_j(a, b)}.$$

2229. *Proposed by Nguyen Xuan Tho, Hanoi University of Science and Technology, Hanoi, Vietnam.*

Show that for every integer $n \geq 1$ with $n \neq 4$,

$$(x^2 + 1)(x^2 + 2) \cdots (x^2 + n) + 1$$

is irreducible in $\mathbb{Z}[x]$.

2230. *Proposed by Joseph L. Pe, Carpentersville, IL.*

We call a set of positive integers *thick* if the set of quotients formed from its elements is dense in the set of positive real numbers $\mathbb{R}^+$

(a) Show that the set of Fibonacci numbers is not thick.
(b) For a fixed positive integer k, show that $\{n^k : n = 1, 2, \ldots\}$ is thick.
(c) Show that the set of primes is thick.

PROBLEMS AND SOLUTIONS

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Solutions to the problems in this issue received through ***Submittable*** by March 15, 2026 will be given full consideration.

PROBLEMS

Editorial comments.

Greg Oman, who has served as *CMJ* Problems editor since 2018, has retired from this position. I am thankful for his valuable service on the board, and particularly for his support and helpful advice when I first began as Editor-Elect of *CMJ*. I know that contributors to *CMJ Problems* have enjoyed working with Greg, and all of us on the *CMJ* board have appreciated his contributions toward the success of this column. We will miss him, and we wish him all the best for his future projects.

I am pleased to announce that William Craig has joined the *CMJ* editorial board as our new Problems editor, beginning with this issue's column. Dr. Craig is currently Assistant Professor of Mathematics at the United States Naval Academy. His journey to interest in research-level mathematics began through competition mathematics in middle school, high school, and finally Putnam competitions. As such, he is quite interested in intriguing problems and their clever solutions!

I have appreciated Greg's and MAA Editorial Coordinator Annie Petitt's assistance as we've made this transition to a new Problems editor. Also much appreciated have been the patience and support from problem authors and referees. I'm excited about this opportunity to work with Will. All of us at *CMJ* look forward to our readers' continued participation in the *CMJ Problems and Solutions* column.

— Tamara Lakins, Editor

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1306. *Proposed by Raymond Mortini, Universit é du Luxembourg, Esch-sur-Alzette, Luxembourg and Rudolf Rupp, Technische Hochschule Nürnberg, Nürnberg, Germany.*

Consider for $x \in (0, 1]$ and for $n \in \mathbb{N} := \{0, 1, 2, \dots\}$ the equation

$$\frac{(1-x)^{n-1}}{x^{2n-1}} = 2. \qquad (*)$$

Prove the following:

(1) For each n there exists a unique $x_n \in (0, 1)$ solving equation $(*)$.
(2) Prove that $L := \lim_{n\to\infty} x_n$ exists.
(3) Determine L.

1307. *Proposed by Himadri Lal Das, Department of Mathematics, Indian Institute of Technology, Kharagpur, India.*

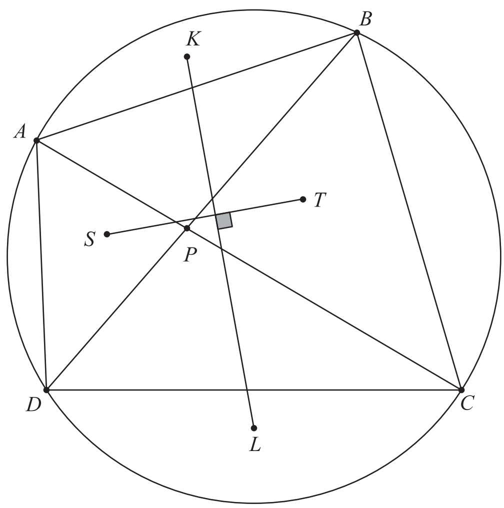
Let $\left[{n \atop k}\right]$ denote the number of permutations of n elements whose decomposition has k cycles and $\binom{n}{k}$ denote the number of k-combinations of n elements. Prove

$$\frac{1}{(p-1)!}\sum_{j=0}^{p}\sum_{k=0}^{n} k^j\left((n+1)\left[{p \atop j+1}\right] - \left[{p \atop j}\right]\right) = \binom{n+p+1}{n}$$

provided $0^0 = 1$ and n and p are positive integers.

1308. *Proposed by Tran Quang Hung, High School for Gifted Students, Vietnam National University, Hanoi, Vietnam.*

Let $ABCD$ be a cyclic quadrilateral. Let P be the intersection of two diagonals AC and BD. Let K and L be the circumcenters of triangles PCD and PAB, respectively. Let S and T be the symmedian points of triangles PAD and PBC, respectively. Prove that the two lines KL and ST are perpendicular. (The symmedian point of a triangle is the intersection of the three symmedian lines: these lines are the reflections of the medians in the corresponding angle bisectors).



1309. *Proposed by Ovidiu Furdui and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.*

Calculate the following expressions and prove your solutions are correct.

(a) $L = \lim_{n\to\infty} \int_0^1 \left(\frac{n}{n+\sqrt[n]{x}}\right)^n dx$

(b) $\lim_{n\to\infty} n \left(\int_0^1 \left(\frac{n}{n+\sqrt[n]{x}}\right)^n dx - L\right)$

1310. *Proposed by Eugen J. Ionaşcu, Columbus State University, Columbus, GA.*

Prove, for each $n \in \mathbb{N}$, that

$$\sum_{k=0}^{n} \frac{(-1)^k \binom{n}{k}}{2k+1} > \frac{1}{2n+1}.$$

PROBLEMS AND SOLUTIONS

Edited by **Daniel H. Ullman, Daniel J. Velleman, Stan Wagon, and Douglas B. West**

with the collaboration of Bonnie Amende, Paul Bracken, Hongwei Chen, Raluca Dumitru, Zachary Franco, George Gilbert, Leonid V. Kovalev, László Lipták, Rick Luttmann, Frank B. Miles, Lenhard Ng, Rajesh Pereira, Kenneth Stolarsky, Richard Stong, Lawrence Washington, and Li Zhou.

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PROBLEMS

12566. *Proposed by Paul Bracken, University of Texas, Edinburg, TX.* Let a_1 be a positive real number. For $n \geq 1$, let $a_{n+1} = a_n + 1/a_n + 2$, and let $b_n = a_n - 2n - (1/2)\ln n$. Compute $\lim_{n\to\infty}(n^2/\ln n)\,(b_{n+1} - b_n)$.

12567. *Proposed by Tsovak Yegoryan, Physics and Mathematics Specialized School named after Artashes Shahinyan, Yerevan, Armenia.* Let n be a positive integer. What is the minimum value of

$$\sum_{i=1}^{k-1} \frac{a_{i+1}}{\gcd(a_i, a_{i+1})},$$

where the minimum is taken over all k and over all strictly increasing sequences $(a_1, \ldots, a_k)$ of integers such that $a_1 = 1$ and $a_k = n$?

12568. *Proposed by Tho Nguyen Xuan, Hanoi University of Science and Technology, Hanoi, Vietnam.* For $r > 0$, what is the maximum value of

$$\frac{|(a-b)(c-a)(b-c)| \cdot |a+b+c|^r}{(a^2+b^2+c^2)^{(3+r)/2}}$$

over real numbers a, b, and c not all equal to zero?

12569. *Proposed by Aryan Desai, Ahmedabad, India, Péter Pál Pálfy, San Francisco, CA, and Martin Sinclair, Boston, MA.* Let n be a positive integer. For a function f with domain $\{0,1\}^n$ and an n-by-n binary matrix A with rows $r_1, \ldots, r_n$, we write $f(A)$ for $\big(f(r_1), \ldots, f(r_n)\big)$. How many functions $f\colon \{0,1\}^n \to \{0,1\}$ have the property that $f(f(A)) = f(f(A^T))$ for all binary n-by-n matrices A?

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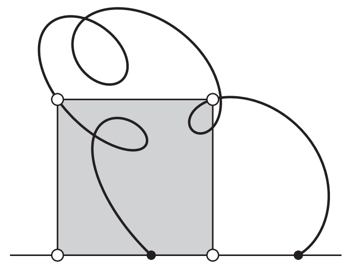
12570. *Proposed by Robert Rogozsan, Baia Mare, Romania.* Let $M_n(F)$ be the set of n-by-n matrices with entries in a field F. Find all symmetric and invertible matrices $A \in M_n(\mathbb{R})$ for which there exists an invertible matrix $B \in M_n(\mathbb{C})$ such that $AB + BA = B\left(A^2 + I\right)$, where I is the n-by-n identity matrix.

12571. *Proposed by Vladimir Bobkov and Timur Galimov, Ufa Federal Research Center of the Russian Academy of Sciences, Ufa, Russia.* Let $a_1, a_2, \ldots$ be a bounded sequence of real numbers with the property that whenever a subsequence $\{a_{n_k}\}$ converges, then, for any natural number p, the shifted subsequence $\{a_{n_k+p}\}$ converges to the same limit. Must $\{a_n\}$ converge as well?

12572. *Proposed by Tibor Beke, University of Massachusetts, Lowell, MA.* Consider a continuous function from $[0, 1]$ to $\mathbb{R}^2$ with coordinates given by $t \mapsto (x(t), y(t))$. Assume that $x(0) < x(1)$, $y(0) = y(1) = 0$, and $y(t) > 0$ for $0 < t < 1$. Prove that there exist $a, b \in (0, 1)$ such that $(x(a), 0)$, $(x(a), y(a))$, $(x(b), 0)$, and $(x(b), y(b))$ form the vertices of a square. The figure shows an example.



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Solutions written by 4 or more authors that are later published as a featured solution to a problem published in this issue will be designated as being written by a problem solving group; if a name for the problem solving group is not submitted by the authors, then one will be chosen by the editors.

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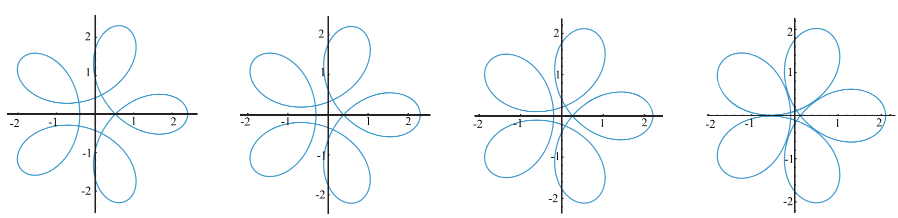
PROBLEMS

1311. *Proposed by Gregory Dresden, Washington & Lee University, Lexington, VA.*

The parametric equation

$$x(t) = B\cos t + \cos 4t$$
$$y(t) = B\sin t \; - \sin 4t$$

is called a *hypotrochoid*. Shown here, from left to right, are the graphs as B decreases from 1.4 down to 1.1.



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Note that as B decreases, the five outer loops of the hypotrochoid approach each other. Find the one value of $B > 1$ such that the five loops of the hypotrochoid are tangent at their point of intersection. (The answer is almost, but not quite, $B = 1.1$.)

1312. *Proposed by Götz Trenkler, TU Dortmund University, Germany, and Dietrich Trenkler, University of Osnabrück, Germany*

Let a, b, and c be linearly independent vectors from $\mathbb{R}^n$ and a^T, b^T, c^T their respective transposes. Show that

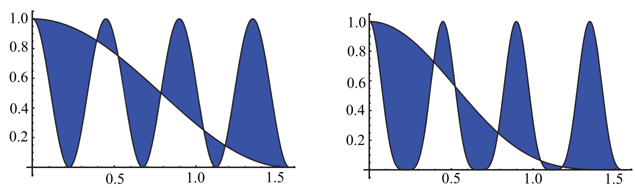
$$(a^T a)(b^T b) + (a^T a)(c^T c) + (b^T b)(c^T c) \leq (a^T a)^2 + (b^T b)^2 + (c^T c)^2 + 3[(a^T b)^2 + (a^T c)^2 + (b^T c)^2].$$

1313. *Proposed by Reza Farhadian, Razi University, Iran.*

Goldbach's conjecture asserts that every even integer $k > 2$ is the sum of two primes. Prove that the probability that Goldbach's conjecture fails for an even integer $2n$ approaches zero as $n \to \infty$.

1314. *Proposed by Gregory Dresden, Washington & Lee University, Lexington, VA.*

On the left is the area between $\cos^2(x)$ and $\cos^2(7x)$ over the interval $[0, \pi/2]$, and on the right is the area between $\cos^4(x)$ and $\cos^4(7x)$ over the same interval.



A careful eye will reveal that the six shaded regions on the left are slightly different from the six shaded regions on the right. However, the total *areas* of the shaded regions (on the left, and on the right) are in fact equal to each other. Show that this is true in general: the total area between $\cos^2(x)$ and $\cos^2(nx)$ is always equal to the total area between $\cos^4(x)$ and $\cos^4(nx)$, for n any odd integer and for the same interval $[0, \pi/2]$.

1315. *Proposed by Reza Farhadian, Razi University, Iran.*

Let A_n be an increasing sequence of positive real numbers satisfying the property $\frac{m}{A_m} \leq \frac{n}{A_n}$ for $m < n$, and satisfying the asymptotic $A_n \sim n \log(A_n)$. Find the limit

$$\lim_{n\to\infty} (A_1 A_2 \cdots A_n)^{\frac{1}{A_n}}.$$

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12573. *Proposed by Dan Dima, Bucharest, Romania, and Peter Winkler, Dartmouth College, Hanover, NH.* A certain bar offers n seats in a line. The first customer to enter can sit anywhere. Subsequent patrons arrive one at a time and are social-distancing; they will leave if all seats are occupied or adjacent to an occupied seat, and otherwise will choose a seat such that the distance from their seat to their nearest neighbor is as large as possible. For which n can the first customer choose a seat that will result in maximal occupancy, in other words, that will result in $\lceil n/2 \rceil$ seats being occupied?

12574. *Proposed by Tuan Anh Nguyen, Nguyen Quang Dieu High School for the Gifted, Dong Thap, Vietnam.* Let $L_1 = 1$, $L_2 = 3$, and $L_m = L_{m-1} + L_{m-2}$ when $m \geq 3$. (These are the *Lucas* numbers.) A *composition* of n is a list of positive integers whose sum is n. For example, the compositions of 4 are (4), (3,1), (1,3), (2,2), (2,1,1), (1,2,1), (1,1,2), and (1,1,1,1). Given a composition c, let $o(c)$ and $e(c)$ be the number of odd parts and even parts, respectively, of c. Prove

$$\sum 3^{o(c)}(-2)^{e(c)} = L_{2n},$$

where the sum is taken over all compositions c of n.

12575. *Proposed by Hüseyin Yiğit Emekçi, Izmir, Turkey, and George T. Gilbert, Texas Christian University, Fort Worth, TX.* Let r be a real number with $r \geq 2$, and let $b_1, \ldots, b_n$ be positive real numbers satisfying $\sum_{i=1}^{n} b_i = n$. Prove

$$\sum_{i=1}^{n} \frac{1}{b_i^r - b_i + n} \leq 1,$$

with equality if and only if $b_i = 1$ for all i.

doi.org/10.1080/00029890.2025.2572927

12576. *Proposed by Erik Vigren, Swedish Institute of Space Physics, Uppsala, Sweden.* For positive integers k and n greater than 2, let $f_k(n)$ be the largest positive integer m such that $m! < (n!)^k$. For example, $f_3(5) = 9$ because $9! < (5!)^3$ but $10! > (5!)^3$.
(a) Prove that if $k \in \{2, 3\}$, then $f_k(n+1) - f_k(n) \in \{k-1, k\}$ for all n.
(b) Prove that if $k \geq 4$, then $f_k(n+1) - f_k(n) \in \{k-1, k\}$ for sufficiently large n.

12577. *Proposed by Haoran Chen, Suzhou, China.* When A, B, and C are points in the plane with $\angle BAC < \pi$, we say that $\angle BAC$ is *well-divided* by a finite set of interior points if, when the interior points are $X_1, \ldots, X_k$ in angular order from B to C, we have $\angle BAX_1 = \angle X_1AX_2 = \cdots = \angle X_kAC$. Suppose that we have n points in the plane, with $n \geq 5$ and with no three of the points collinear, and suppose that any angle formed by three of the points is well divided by the other points (if any) that lie inside the angle. Prove that the points form a regular n-gon.

12578. *Proposed by Navid Safaei, Bulgarian Academy of Sciences, Sofia, Bulgaria.* Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that $f(x + 4f(y)) = f(x + 3y) + f(y)$ for all positive real numbers x and y.

12579. *Proposed by Andrei Vila, International Computer High School of Bucharest, Romania, and Robert Rogozsan, Baia Mare, Romania.* Let A and B be n-by-n complex matrices with $n \geq 3$ such that $AB = BA^{n-1} + I$, where I is the n-by-n identity matrix. Prove that A is invertible.