

# Description of the Math Accuplacer Test

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**Union Building Testing Center Room 323:** 801-626-7945

**Davis Campus Testing Center Room 215:** 801-395-3495

For other locations, schedules and additional info go to [www.weber.edu/TestingCenter/accpl.html](http://www.weber.edu/TestingCenter/accpl.html)

**Cost:** Math: \$10.00 (Score Reprints: \$10.00)

**Bring to Testing Center:** Picture ID and W#. Calculators are NOT allowed but scratch paper will be provided.

**Accuplacer may be taken twice a year.** If you feel you will not place into a QL math course or MATH 1060 Trigonometry please take the Math Mastery Placement Exam. You may not place into MATH 1010 or below with the Accuplacer.

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## Arithmetic Portion

**17 questions**

**120 point maximum**

- **Operations with whole numbers and fractions** – addition, subtraction, multiplication, division, recognizing equivalent fractions and mixed numbers.
  - **Operations with decimals and percents** – additions, subtraction, multiplication, and division percent problems, decimal recognition, fraction percent equivalencies, and estimation problems.
  - **Applications and problem solving** – rate, percent, and measurement problems, geometry problems, distribution of a quantity into its fractional parts.
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## Elementary Algebra Portion

**12 questions**

**120 point maximum**

- **Operations with integers and rational numbers** – computation with integers and negative rationals, the use of absolute values, and ordering.
  - **Operations with algebraic expressions** – evaluations of simple formulas, expressions, and adding, subtracting monomials and polynomials, the evaluation of positive rational roots and exponents, simplifying algebraic fractions, and factoring.
  - **Equations solving, inequalities, and word problems** – solving verbal problems presented in algebraic context, geometric reasoning, the translation of written phrases into algebraic expressions, graphing.
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## College Level Math Portion

**20 questions**

**120 point maximum**

1. **CLM Score of 50 to 69** places student in any QL Math course (MATH 1030 Contemporary Math, 1040 Intro to Statistics, 1050 College Algebra, or 1080 Precalculus) or MATH 1060 Trigonometry.
  2. **CLM Score of 70 to 89** fulfills QL requirement and student is able to register for courses that require MATH 1050 as a prerequisite (MATH 1050 College Algebra will not appear on transcript and no credits are earned.)
  3. **CLM Score of 90 or higher** places student in MATH 1210 Calculus I.
- **Algebraic operations** – simplifying rational algebraic expressions, factoring and expanding polynomials, manipulating roots and exponents.
  - **Solutions of equations and inequalities** – the solution of linear and quadratic equations by factoring, expanding polynomials, manipulating roots and exponents.
  - **Coordinate geometry** – plane geometry, the coordinate plane, straight lines, conics, sets of points in a plane, graphs of algebraic functions.
  - **Application and other algebra topics** – complex numbers, series and sequences, determinants, permutations, combinations, fractions, word problems.
  - **Functions and trigonometry** – polynomial, algebraic, exponential, logarithmic, trigonometric functions.

# CENTERS

## Distance Accuplacer Proctor Request

### To take the Accuplacer in your local community:

- 1. You must be far enough away from Weber State University to qualify for the Distance Accuplacer.**  
Outside of Utah qualifies, but inside Utah the test will not be sent to any site between the Utah/Idaho state line and the Point of the Mountain with the exception of Utah State University in Logan, UT.
- 2. Contact a local college or university testing center ( High Schools are not acceptable ).**
  - Ask if they will proctor your assessment test.
  - The testing center will need a computer with Internet access.
  - Be sure the chosen proctor is willing to give a test lasting anywhere from 1-2-+ hours.
- 3. Register for your desired test (buttons in right-column).**
- 4. Your request will be automatically emailed to our office.**  
Your test request can take 2 to 7 business days to process. We will notify the proctor with the needed information for your assessment test.
- 5. There is a \$10.00 fee per test, payable to Weber State University.**  
This fee must be paid before the test can be setup. Once the test is sent to the proctor the payment is processed and no refund is given.
- 6. You will be notified by email once your test has been sent to the proctor.**
- 7. After you have been notified that your placement test has been sent to the proctor, you will have until the end of the month to take your test.**  
After that time the information sent to the proctor will no longer be valid.

Distance Accuplacer Proctor Request Website:

<http://www.weber.edu/TestingCenter/distance-accuplacer.html>

## Arithmetic Sample Questions

*This test measures your ability to perform basic arithmetic operations and to solve problems that involve fundamental arithmetic concepts. There are 17 questions on the Arithmetic tests, divided into three types.*

*• Operations with whole numbers and fractions: Topics included in this category are addition, subtraction, multiplication, division, recognizing equivalent fractions and mixed numbers, and estimating.*

*• Operations with decimals and percents: Topics include addition, subtraction, multiplication, and division with decimals. Percent problems, recognition of decimals, fraction and percent equivalencies, and problems involving estimation are also given.*

*• Applications and problem solving: Topics include rate, percent, and measurement problems; simple geometry problems; and distribution of a quantity into its fractional parts.*

1.  $2.75 + .003 + .158 =$

- A. 4.36
- B. 2.911
- C. 0.436
- D. 2.938

2.  $7.86 \times 4.6 =$

- A. 36.156
- B. 36.216
- C. 351.56
- D. 361.56

3.  $\frac{7}{20} =$

- A. .035
- B. 0.858
- C. 0.35
- D. 3.5

4. Which of the following is the least?

- A. 0.105
- B. 0.501
- C. 0.015
- D. 0.15

5. All of the following are ways to write 25 percent of N EXCEPT:

- A.  $0.25 N$
- B.  $25N/100$
- C.  $1/4 N$
- D.  $25 N$

6. Which of the following is closest to  $27.8 \times 9.6$ ?

- A. 280
- B. 300
- C. 2,800
- D. 3,000

7. A soccer team played 160 games and won 65% of them. How many games did it win?

- A. 94
- B. 104
- C. 114
- D. 124

8. 32 is 40% of what number?

- A. 12.8
- B. 128
- C. 80
- D. 800

**Arithmetic – cont'**

9. Three people who work full-time are to work together on a project, but their total time on the project is to be equivalent to that of only one person working full-time. If one of the people is budgeted for one-half of his time to the project and a second person for one-third of her time, what part of the third worker's time should be budgeted to this project?

- A.  $1/3$
- B.  $3/5$
- C.  $1/6$
- D.  $1/8$

10.  $3\frac{1}{3} - 2\frac{2}{5} =$

- A.  $1\frac{1}{2}$
- B.  $1/15$
- C.  $14/15$
- D.  $1\frac{1}{15}$

## Elementary Algebra Sample Questions

*A total of 12 questions of three types are administered in this test.*

- The first type involves operations with integers and rational numbers, and includes computation with integers and negative rationals, the use of absolute values, and ordering.*
- The second type involves operations with algebraic expressions using evaluation of simple formulas and expressions, and adding and subtracting monomials and polynomials. Questions involve multiplying and dividing monomials and polynomials, the evaluation of positive rational roots and exponents, simplifying algebraic fractions, and factoring.*
- The third type of question involves translating written phrases into algebraic expressions and solving equations, inequalities, word problems, linear equations and inequalities, quadratic equations (by factoring), and verbal problems presented in an algebraic context.*

1. If A represents the number of apples purchased at 15 cents each, and B represents the number of bananas purchased at 10 cents each, which of the following represents the total value of the purchases in cents?

- A.  $A+B$
- B.  $25(A+B)$
- C.  $10A + 15B$
- D.  $15A + 10B$

2.  $\sqrt{2} \times \sqrt{15} =$

- A. 17
- B. 30
- C.  $\sqrt{30}$
- D.  $\sqrt{17}$

*Elementary Algebra – cont'*

3. What is the value of the expression  $2x^2 + 3xy - 4y^2$  when  $x = 2$  and  $y = -4$  ?

- A. -80
- B. 80
- C. -32
- D. 32

4. In the figure below, both circles have the same center, and the radius of the larger circle is  $R$ . If the radius of the smaller circle is 3 units less than  $R$ , which of the following represents the area of the shaded region?



- A.  $\pi R^2$
- B.  $\pi(R-3)^2$
- C.  $\pi R^2 - \pi \times 3^2$
- D.  $\pi R^2 - \pi(R-3)^2$

5.  $(3x - 2y)^2 =$

- A.  $9x^2 - 4y^2$
- B.  $9x^2 + 4y^2$
- C.  $9x^2 + 4y^2 - 6xy$
- D.  $9x^2 + 4y^2 - 12xy$

6. If  $x > 2$ , then  $\frac{x^2 - x - 6}{x^2 - 4} =$

- A.  $\frac{x-3}{2}$
- B.  $\frac{x-3}{x-2}$
- C.  $\frac{x-3}{x+2}$
- D.  $\frac{3}{2}$

7. If  $x \neq 2$ , then  $\frac{4 - (-6)}{-5} =$

- A.  $\frac{2}{5}$
- B.  $-\frac{2}{5}$
- C. 2
- D. -2

8. If  $2x - 3(x+4) = -5$ , then  $x =$

- A. 7
- B. -7
- C. 17
- D. -17

9.  $-3(5 - 6) - 4(2 - 3) =$

- A. -7
- B. 7
- C. -1
- D. 1

10. Which of the following expressions is equivalent to  $20 - \frac{4}{5}x \geq 16$  ?

- A.  $x \leq 5$
- B.  $x \geq 5$
- C.  $x \geq 32\frac{1}{2}$
- D.  $x \leq 32\frac{1}{2}$

## College Level Math Sample Questions

The College-Level Mathematics test measures your ability to solve problems that involve college-level mathematics concepts. There are six content areas measured on this test: (a) Algebraic Operations, (b) Solutions of Equations and Inequalities, (c) Coordinate Geometry, (d) Applications and other Algebra Topics, (e) Functions, and (f) Trigonometry. The Algebraic Operations content area includes the simplification of rational algebraic expressions, factoring and expanding polynomials, and manipulating roots and exponents. The Solutions of Equations and Inequalities content area includes the solution of linear and quadratic equations and inequalities, systems of equations, and other algebraic equations. The Coordinate Geometry content area presents questions involving plane geometry, the coordinate plane, straight lines, conics, sets of points in the plane, and graphs of algebraic functions. The Functions content area includes questions involving polynomial, algebraic, exponential, and logarithmic functions. The Trigonometry content area includes trigonometric functions. The Applications and other Algebra Topics content area contains complex numbers, series and sequences, determinants, permutations and combinations, factorials, and word problems. A total of 20 questions are administered on this test.

1.  $2^{5/2} - 2^{3/2} =$

- A.  $2^{1/2}$
- B. 2
- C.  $2^{3/2}$
- D.  $2^{5/3}$
- E.  $2^2$

2. If  $a \neq b$ , and  $\frac{1}{x} + \frac{1}{a} = \frac{1}{b}$ , then  $x =$

- A.  $\frac{1}{b} - \frac{1}{a}$
- B.  $b - a$
- C.  $\frac{1}{ab}$
- D.  $\frac{a-b}{ab}$
- E.  $\frac{ab}{a-b}$

3. If  $3x^2 - 2x + 7 = 0$ , then  $(x - \frac{1}{3})^2 =$

- A.  $\frac{20}{9}$
- B.  $\frac{7}{9}$
- C.  $-\frac{7}{9}$
- D.  $-\frac{8}{9}$
- E.  $-\frac{20}{9}$

4. The graph of which of the following equations is a straight line parallel to the graph of  $y = 2x$ ?

- A.  $4x - y = 4$
- B.  $2x - 2y = 2$
- C.  $2x - y = 4$
- D.  $2x + y = 2$
- E.  $x - 2y = 4$

5. An equation of the line that contains the origin and the point (1,2) is

- A.  $y = 2x$
- B.  $2y = x$
- C.  $y = x - 1$
- D.  $y = 2x + 1$
- E.  $\frac{y}{2} = x - 1$

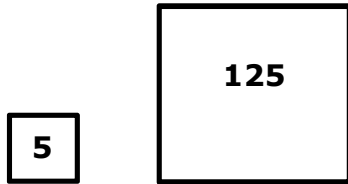
6. An apartment building contains 12 units consisting of one- and two-bedroom apartments that rent for \$360 and \$450 per month, respectively. When all units are rented, the total monthly rental is \$4,950. What is the number of two-bedroom apartments?

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

College Level Math – cont'

7. If the two square regions in the figures below have the respective areas indicated in square yards, how many yards of fencing are needed to enclose the two regions?

- A.  $4\sqrt{130}$
- B.  $20\sqrt{10}$
- C.  $24\sqrt{5}$
- D. 100
- E.  $104\sqrt{5}$



8. If  $\log_{10} x = 3$ , then  $x =$

- A.  $3^{10}$
- B. 1,000
- C. 30
- D.  $\frac{10}{3}$
- E.  $\frac{3}{10}$

9. If  $f(x) = 2x + 1$  and  $g(x) = \frac{x-1}{2}$ , then  $f(g(x)) =$

- A.  $x$
- B.  $\frac{x-1}{4x+2}$
- C.  $\frac{4x+2}{x-1}$
- D.  $\frac{5x+1}{2}$
- E.  $\frac{(2x+1)(x-1)}{2}$

10. If  $\theta$  is an acute angle and  $\sin \theta = \frac{1}{2}$ , then  $\cos \theta =$

- A. -1
- B. 0
- C.  $\frac{1}{2}$
- D.  $\frac{\sqrt{3}}{2}$
- E. 2

Arithmetic: 1) B 2) A 3) C 4) C 5) D 6) A 7) B 8) C 9) C 10) C  
Elementary Algebra: 1) D 2) C 3) A 4) D 5) D 6) B 7) D 8) B 9) B 10) A  
College Level Math: 1) C 2) E 3) E 4) C 5) A 6) E 7) C 8) B 9) A 10) D

# Fraction Rules

## Special Fractions

1.  $\frac{b}{1}$  simplifies to  $b$ .
2.  $\frac{1}{b}$  does not simplify any further.
3.  $\frac{0}{b}$  simplifies to 0.
4.  $\frac{b}{0}$  is undefined.

## Negative Fractions

1.  $-\frac{a}{b}$  is the same as  $\frac{-a}{b}$  and  $\frac{a}{-b}$
2.  $\frac{-a}{-b}$  simplifies to  $\frac{a}{b}$
3.  $-\frac{a}{b}$  is NOT the same as  $\frac{-a}{-b}$

## Addition

1.  $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$
2.  $a + \frac{b}{c} = \frac{ac}{c} + \frac{b}{c} = \frac{ac+b}{c}$
3.  $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$

## Subtraction

1.  $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$
2.  $a - \frac{b}{c} = \frac{ac}{c} - \frac{b}{c} = \frac{ac-b}{c}$
3.  $\frac{a}{b} - c = \frac{a}{b} - \frac{bc}{b} = \frac{a-bc}{b}$
4.  $\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad-bc}{bd}$

## Multiplication

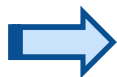
1.  $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
2.  $a \cdot \frac{b}{c} = \frac{a}{1} \cdot \frac{b}{c} = \frac{ab}{c}$
3.  $\frac{a}{b} \cdot c = \frac{a}{b} \cdot \frac{c}{1} = \frac{ac}{b}$

## Division

1.  $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$
2.  $\frac{\frac{a}{b}}{c} = \frac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc}$
3.  $\frac{a}{\frac{b}{c}} = \frac{\frac{a}{1}}{\frac{b}{c}} = \frac{a}{1} \cdot \frac{c}{b} = \frac{ac}{b}$

## Cancellation ( $a \neq 0, b \neq 0, c \neq 0$ )

1.  $\frac{a}{a}$  cancels to 1
2.  $\frac{ab}{ac}$  cancels to  $\frac{b}{c}$
3.  $\frac{a}{b} \cdot \frac{b}{c}$  cancels to  $\frac{a}{c}$



4.  $\frac{a}{b} \cdot \frac{c}{a}$  cancels to  $\frac{c}{b}$
5.  $a \cdot \frac{b}{a}$  cancels to  $b$
6.  $\frac{b}{a} \cdot a$  cancels to  $b$



# Decimal Rules

## Comparing

1. Give the decimals the same number of places. (You can put zeros to the right of a decimal without changing its value.)
2. Compare each number from the decimal to the right, deciding which decimal is larger.

## Addition

1. Line up the decimals with point under point.
2. Add each column and bring the decimal point straight down into each answer. Carry if necessary. You can carry to the left of the decimal point.

$$\begin{array}{r} 12.924 \\ + 3.600 \\ \hline 16.524 \end{array}$$

*Warning: Don't confuse the period at the end of a sentence with the decimal point.*

## Subtraction

1. Use zeros to give each number the same number of decimal places. Compare the decimals to decide which is larger.
2. Line up the numbers with point under point. Be sure to put the larger number on top.
3. Subtract, borrowing if necessary. Bring the decimal point straight down to the answer.

$$\begin{array}{r} 18.200 \\ - 6.008 \\ \hline 12.192 \end{array}$$

## Division by a Decimal

*To divide a number by a decimal, you must first change the divisor to a whole number.*

1. Make the divisor a whole number by moving the point to the right as far as it will go.
2. Move the point in the dividend to the right the same number of places you moved the point in the divisor. You may need to add zeros to the dividend.

$$.45 \overline{) .900}$$

$$45 \overline{) 90.0}$$

3. Bring the point up in the quotient directly above its new position in the dividend and divide.

$$\begin{array}{r} 2.0 \\ 45 \overline{) 90.0} \end{array}$$

90 divided by 45 = 2  
is the same as .900 divided by .45 = 2

## Division of Decimal by Whole Number

1. Put the point in the quotient directly above its position in the dividend.
2. Divide as you would for whole numbers.

## Decimal Rules – cont.

### Multiplication

1. Place one number under the other, lined up on the right side for easy multiplication. Multiply.
2. Count the number of decimal places in both numbers. (Decimal places are to the right of the decimal point.)
3. Counting from the right to the left put the total number of decimal places in your answer. Use zeros if you need more places than you have numbers.

$$37.7 \times 2.8 = ? \text{ --->}$$

$$\begin{array}{r} 37.7 \quad (1 \text{ decimal place}) \\ \times 2.8 \quad (1 \text{ decimal place}) \\ \hline 3016 \\ +754 \\ \hline 105.56 \quad (2 \text{ decimal places, move point 2 places left}) \end{array}$$

### Rounding

1. Underline the digit in the place you are rounding off to.
2. If the digit to the right is 5 or more, add 1 to the underlined digit.
3. If the digit to the right is less than 5, leave the underlined digit as it is.
4. Drop the digits to the right of the underlined digit.

*1.19 to the nearest tenth gives 1.2 (1.20).*

*1.545 to the nearest hundredth gives 1.55.*

*0.1024 to the nearest thousandth gives 0.102.*

*1.80 to the nearest one gives 2.*

*150.090 to the nearest hundred gives 200.*

*4499 to the nearest thousand gives 4000.*

### Estimating

Use estimating to help you check the placement of the decimal point. You could round 37.7 to 40 and 2.8 to 3. It's easy to multiply 3 x 40 so you know your answer should be close to 120.

*Here's a "mental math" shortcut: When multiplying a number by a multiple of ten, just move the decimal point one space to the right for every zero.*

*10 x 0.6284 = 6.284 (1 zero, 1 space right)*

*100 x 0.6284 = 62.84 (2 zeroes, 2 spaces right)*

*1000 x 0.6284 = 628.4 (3 zeroes, 3 spaces right)*

*10,000 x 0.6284 = 6284 (4 zeroes, 4 spaces right)*

*100,000 x 0.6284 = 62,840 (5 zeroes, 5 spaces right)*

# Converting Fractions/Decimals/Percents Rules

## A fraction to a decimal:

Divide the denominator (the bottom part) into the numerator (the top part):

$$\frac{1}{4} = 1 \div 4.00 = 0.25$$

## A percent to a decimal:

Move the decimal point two places to the left. Then, drop the percent sign.

$$25\% = 0.25$$

## A fraction to a percent:

Multiply the fraction by 100 and reduce it. Then, attach a percent sign.

$$\frac{1}{4} \times \frac{100}{1} = \frac{100}{4} = \frac{25}{1} = 25\%$$

## A decimal to a percent:

Move the decimal point two places to the right. Then, attach a percent sign.

$$0.25 = 25\%$$

## A decimal to a fraction:

Starting from the decimal point, count the decimal places. If there is one decimal place, put the number over 10 and reduce. If there are two places, put the number over 100 and reduce. If there are three places, put it over 1000 and reduce, and so on.

$$0.25 = \frac{25}{100} = \frac{1}{4}$$

## A percent to a fraction:

Put the number over 100 and reduce. Then, drop the percent sign.

$$25\% = \frac{25}{100} = \frac{1}{4}$$

## Percent Equation:

What percent of the total is the part?  $\% \times T = P$

12% of the 200 students enrolled in freshman English earned a grade of "A" in the class. How many students earned an "A"?

Ex: 12% of 200 is what - Translate into an equation ("of" means "multiply"; "is" means "equal")

$$0.12 \times 200 = x \quad (\text{Change } \% \text{ to a decimal})$$

$$x = 24 \text{ students earned an A}$$

## Percent Decrease/Increase:

Last year student employment jobs paid \$7.25 per hour. This year student employment jobs are paying \$8.45 per hour. What percent increase was given to student employment jobs?

1. Find the amount of the **increase**:  $\$8.45 - \$7.25 = \$1.20$

2. Which (hourly pay) **total** received an increase? The **\$7.25** per hour got increased.

3. What % of the total was the increase?

$$x \cdot 7.25 = 1.20 \quad \text{or} \quad 7.25x = 1.20$$

$$x = .1655 \quad \text{or} \quad x = 16.6\% \text{ increase}$$

## Place Value

$$10^1 = 10 \quad 10^{-1} = 0.1$$

$$10^2 = 100 \quad 10^{-2} = 0.01$$

$$10^3 = 1000 \quad 10^{-3} = 0.001$$

$$10^4 = 10000 \quad 10^{-4} = 0.0001$$

# Negative and Positive Signs

## Adding Rules:

Positive + Positive = Positive:  $5 + 4 = 9$

Negative + Negative = Negative:  $(-7) + (-2) = -9$

Sum of a negative and a positive number: Use the sign of the larger number and subtract

$$(-7) + 4 = -3$$

$$6 + (-9) = -3$$

$$(-3) + 7 = 4$$

$$5 + (-3) = 2$$

## Subtracting Rules:

Negative - Positive = Negative:

$$(-5) - 3 = -5 + (-3) = -8$$

Positive - Negative = Positive + Positive = Positive:

$$5 - (-3) = 5 + 3 = 8$$

Negative - Negative = Negative + Positive = Use the sign of the larger number and subtract (*Change double negatives to a positive*)

$$(-5) - (-3) = (-5) + 3 = -2$$

$$(-3) - (-5) = (-3) + 5 = 2$$

## Multiplying Rules:

Positive x Positive = Positive:  $3 \times 2 = 6$

Negative x Negative = Positive:  $(-2) \times (-8) = 16$

Negative x Positive = Negative:  $(-3) \times 4 = -12$

Positive x Negative = Negative:  $3 \times (-4) = -12$

## Dividing Rules:

Positive ÷ Positive = Positive:  $12 \div 3 = 4$

Negative ÷ Negative = Positive:  $(-12) \div (-3) = 4$

Negative ÷ Positive = Negative:  $(-12) \div 3 = -4$

Positive ÷ Negative = Negative:  $12 \div (-3) = -4$

# Order of Operations

*In Mathematics, the order in which mathematical problems are solved is extremely important.*

## Rules

1. Calculations must be done from left to right.
2. Calculations in brackets (parenthesis) are done first. When you have more than one set of brackets, do the inner brackets first.
3. Exponents (or radicals) must be done next.
4. Multiply and divide in the order the operations occur.
5. Add and subtract in the order the operations occur.

## Acronyms to Help You Remember

Please **E**xcuse **M**y **D**ear **A**unt **S**ally (**P**arenthesis, **E**xponents, **M**ultiply, **D**ivide, **A**dd, **S**ubtract)

**B**EDMAS (**B**rackets, **E**xponents, **D**ivide, **M**ultiply, **A**dd, **S**ubtract)

**B**ig **E**lephants **D**estroy **M**ice **A**nd **S**nails (**B**rackets, **E**xponents, **D**ivide, **M**ultiply, **A**dd, **S**ubtract)

**P**ink **E**lephants **D**estroy **M**ice **A**nd **S**nails (**P**arenthesis, **E**xponents, **D**ivide, **M**ultiply, **A**dd, **S**ubtract)

# Definitions for Properties of Mathematics

## Distributive Property

The sum of two numbers times a third number is equal to the sum of each addend times the third number.

For example  $a \times (b + c) = a \times b + a \times c$

## Associative Property of Addition

When three or more numbers are added, the sum is the same regardless of the grouping of the addends.

For example  $(a + b) + c = a + (b + c)$

## Associative Property of Multiplication

When three or more numbers are multiplied, the product is the same regardless of the order of the multiplicands.

For example  $(a \times b) \times c = a \times (b \times c)$

## Commutative Property of Addition

When two numbers are added, the sum is the same regardless of the order of the addends.

For example  $a + b = b + a$

## Commutative Property of Multiplication

When two numbers are multiplied together, the product is the same regardless of the order of the multiplicands.

For example  $a \times b = b \times a$

## Identity Property of Addition

The sum of any number and zero is the original number.

For example  $a + 0 = a$ .

## Identity Property of Multiplication

The product of any number and one is that number.

For example  $a \times 1 = a$ .

## Additive Inverse of a Number

The additive inverse of a number,  $a$  is  $-a$  so that  $a + (-a) = 0$ .

## Multiplicative Inverse of a Number

The multiplicative inverse of a number,  $a$  is  $\frac{1}{a}$  so that  $a \times \frac{1}{a} = 1$

## Multiplication Property of Zero

Multiplying any number by 0 yields 0.

For example  $a \times 0 = 0$ .

## Addition Property of Zero

Adding 0 to any number leaves it unchanged.

For example  $a + 0 = a$ .

## Property of Equality for Addition

Property of Equality for Addition says that if  $a = b$ , then  $a + c = b + c$ .

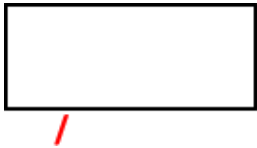
If you add the same number to both sides of an equation, the equation is still true

## Property of Equality for Multiplication

Property of Equality for Multiplication says that if  $a = b$ , then  $a \times c = b \times c$ .

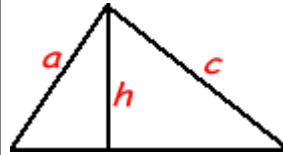
If you multiply the same number to both sides of an equation, the equation is still true.

# Geometry



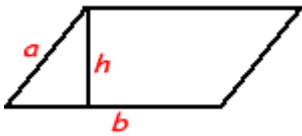
## Rectangle

Area = Length X Width  $A = lw$   
Perimeter = 2 X Lengths + 2 X Widths  $P = 2l + 2w$



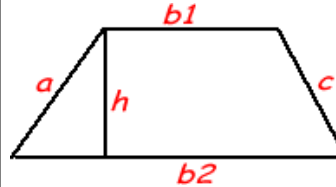
## Triangle

Area = 1/2 of the base X the height  $A = 1/2 bh$   
Perimeter =  $a + b + c$  (add the length of the 3 sides)



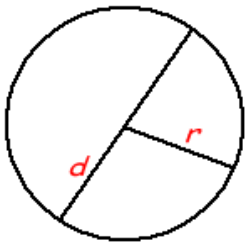
## Parallelogram

Area = Base X Height  $A = bh$   
Perimeter = 2 X Lengths + 2 X Widths  $P = 2l + 2w$



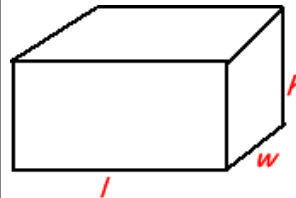
## Trapezoid

$A = \left(\frac{b1 + b2}{2}\right)h$   
Perimeter =  $a + b1 + b2 + c$



## Circle

The distance around the circle is a circumference. The distance across the circle is the diameter (d). The radius (r) is the distance from the center to a point on the circle. (Pi = 3.14)  
 $d = 2r$   $c = \pi d = 2 \pi r$   $A = \pi r^2$



## Rectangular Solid

Volume = Length X Width X Height  $V = lwh$   
Surface =  $2lw + 2lh + 2wh$

## Angles

Right Angles: 90 degrees

Straight Angle: 180 degrees

Complementary Angles: Two angles the sum of whose measures is 90 degrees

Supplementary Angles: Two angles the sum of whose measures is 180 degrees

## Triangles

Triangles: Sum of the interior angles is 180 degrees

Isosceles Triangle: Two equal sides; two equal angles

Equilateral Triangle: Three equal sides; three equal angles

Right Triangles - Pythagorean Theorem:

$a^2 + b^2 = c^2$ , where  $a$  and  $b$  are the measures of the legs of the triangle and  $c$  is the hypotenuse.

# Ratio and Proportion

## Ratio

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:). Suppose we want to write the ratio of 8 and 12. We can write this as 8:12 or as a fraction  $\frac{8}{12}$ , and we say the ratio is *eight to twelve*.

### Examples:

Jeannine has a bag with 3 videocassettes, 4 marbles, 7 books, and 1 orange.

1) What is the ratio of books to marbles?  
Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be  $\frac{7}{4}$ .  
Two other ways of writing the ratio are 7 to 4, and 7:4.

2) What is the ratio of videocassettes to the total number of items in the bag?  
There are 3 videocassettes, and  $3 + 4 + 7 + 1 = 15$  items total.  
The answer can be expressed as  $\frac{3}{15}$ , 3 to 15, or 3:15.

## Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

### Example:

Are the ratios 3 to 4 and 6:8 equal?

The ratios are equal if  $\frac{3}{4} = \frac{6}{8}$ .

These are equal if their cross products are equal; that is, if  $3 \times 8 = 4 \times 6$ . Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters!

A ratio of 1:7 is not the same as a ratio of 7:1.

### Examples:

Are the ratios 7:1 and 4:81 equal? No!

$\frac{7}{1} > 1$ , but  $\frac{4}{81} < 1$ , so the ratios can't be equal.

Are 7:14 and 36:72 equal?

Notice that  $\frac{7}{14}$  and  $\frac{36}{72}$  are both equal to  $\frac{1}{2}$ , so the two ratios are equal.

## Proportion

A proportion is an equation with a ratio on each side.

It is a statement that two ratios are equal.

$\frac{3}{4} = \frac{6}{8}$  is an example of a proportion.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Question marks or letters are frequently used in place of the unknown number.

### Example:

Solve for  $n$ :  $\frac{1}{2} = \frac{n}{4}$ .

Using cross products we see that  $2 \times n = 1 \times 4 = 4$ , so  $2 \times n = 4$ . Dividing both sides by 2,  $n = 4 \div 2$  so that  $n = 2$ .

# Algebra

## Basic Properties & Facts

### Arithmetic Operations

$$ab + ac = a(b + c) \quad a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\left(\frac{a}{b}\right) \frac{c}{d} = \frac{ac}{bd} \quad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c} \quad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab+ac}{a} = b+c, \quad a \neq 0 \quad \left(\frac{a}{b}\right) \left(\frac{c}{d}\right) = \frac{ac}{bd}$$

### Exponent Properties

$$a^n a^m = a^{n+m} \quad \frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$(a^n)^m = a^{nm} \quad a^0 = 1, \quad a \neq 0$$

$$(ab)^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n} \quad \frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} \quad a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(a^n\right)^{\frac{1}{m}}$$

### Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^n} = a, \text{ if } n \text{ is odd}$$

$$\sqrt[n]{a^n} = |a|, \text{ if } n \text{ is even}$$

### Properties of Inequalities

If  $a < b$  then  $a + c < b + c$  and  $a - c < b - c$

If  $a < b$  and  $c > 0$  then  $ac < bc$  and  $\frac{a}{c} < \frac{b}{c}$

If  $a < b$  and  $c < 0$  then  $ac > bc$  and  $\frac{a}{c} > \frac{b}{c}$

### Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$$|a| \geq 0 \quad |-a| = |a|$$

$$|ab| = |a||b| \quad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$|a + b| \leq |a| + |b| \quad \text{Triangle Inequality}$$

### Distance Formula

If  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$  are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### Complex Numbers

$$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, \quad a \geq 0$$

$$(a + bi) + (c + di) = a + c + (b + d)i$$

$$(a + bi) - (c + di) = a - c + (b - d)i$$

$$(a + bi)(c + di) = ac - bd + (ad + bc)i$$

$$(a + bi)(a - bi) = a^2 + b^2$$

$$|a + bi| = \sqrt{a^2 + b^2} \quad \text{Complex Modulus}$$

$$\overline{(a + bi)} = a - bi \quad \text{Complex Conjugate}$$

$$\overline{(a + bi)}(a + bi) = |a + bi|^2$$



## Logarithms and Log Properties

### Definition

$y = \log_b x$  is equivalent to  $x = b^y$

### Example

$\log_5 125 = 3$  because  $5^3 = 125$

### Special Logarithms

$\ln x = \log_e x$  natural log

$\log x = \log_{10} x$  common log

where  $e = 2.718281828\mathbf{K}$

### Logarithm Properties

$$\log_b b = 1 \quad \log_b 1 = 0$$

$$\log_b b^x = x \quad b^{\log_b x} = x$$

$$\log_b (x^r) = r \log_b x$$

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y$$

The domain of  $\log_b x$  is  $x > 0$

## Factoring and Solving

### Factoring Formulas

$$x^2 - a^2 = (x+a)(x-a)$$

$$x^2 + 2ax + a^2 = (x+a)^2$$

$$x^2 - 2ax + a^2 = (x-a)^2$$

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$$

$$x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^3$$

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$

$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$

If  $n$  is odd then,

$$x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + \mathbf{L} + a^{n-1})$$

$$x^n + a^n$$

$$= (x+a)(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \mathbf{L} + a^{n-1})$$

### Quadratic Formula

Solve  $ax^2 + bx + c = 0$ ,  $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If  $b^2 - 4ac > 0$  - Two real unequal solns.

If  $b^2 - 4ac = 0$  - Repeated real solution.

If  $b^2 - 4ac < 0$  - Two complex solutions.

### Square Root Property

If  $x^2 = p$  then  $x = \pm\sqrt{p}$

### Absolute Value Equations/Inequalities

If  $b$  is a positive number

$$|p| = b \Rightarrow p = -b \text{ or } p = b$$

$$|p| < b \Rightarrow -b < p < b$$

$$|p| > b \Rightarrow p < -b \text{ or } p > b$$

### Completing the Square

Solve  $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the  $x^2$

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of  $x$ , square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for  $x$

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

## Functions and Graphs

### Constant Function

$$y = a \quad \text{or} \quad f(x) = a$$

Graph is a horizontal line passing through the point  $(0, a)$ .

### Line/Linear Function

$$y = mx + b \quad \text{or} \quad f(x) = mx + b$$

Graph is a line with point  $(0, b)$  and slope  $m$ .

#### Slope

Slope of the line containing the two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

#### Slope – intercept form

The equation of the line with slope  $m$  and y-intercept  $(0, b)$  is

$$y = mx + b$$

#### Point – Slope form

The equation of the line with slope  $m$  and passing through the point  $(x_1, y_1)$  is

$$y = y_1 + m(x - x_1)$$

### Parabola/Quadratic Function

$$y = a(x - h)^2 + k \quad f(x) = a(x - h)^2 + k$$

The graph is a parabola that opens up if  $a > 0$  or down if  $a < 0$  and has a vertex at  $(h, k)$ .

### Parabola/Quadratic Function

$$y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$$

The graph is a parabola that opens up if  $a > 0$  or down if  $a < 0$  and has a vertex at  $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ .

### Parabola/Quadratic Function

$$x = ay^2 + by + c \quad g(y) = ay^2 + by + c$$

The graph is a parabola that opens right if  $a > 0$  or left if  $a < 0$  and has a vertex at  $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$ .

### Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Graph is a circle with radius  $r$  and center  $(h, k)$ .

### Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Graph is an ellipse with center  $(h, k)$  with vertices  $a$  units right/left from the center and vertices  $b$  units up/down from the center.

### Hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at  $(h, k)$ , vertices  $a$  units left/right of center and asymptotes that pass through center with slope  $\pm \frac{b}{a}$ .

### Hyperbola

$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at  $(h, k)$ , vertices  $b$  units up/down from the center and asymptotes that pass through center with slope  $\pm \frac{b}{a}$ .

## Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$ , $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2 x^2 x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{a+bx}{a} \neq 1+bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \neq -ax-a$	$-a(x-1) = -ax+a$ Make sure you distribute the “-“!
$(x+a)^2 \neq x^2+a^2$	$(x+a)^2 = (x+a)(x+a) = x^2+2ax+a^2$
$\sqrt{x^2+a^2} \neq x+a$	$5 = \sqrt{25} = \sqrt{3^2+4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3+4=7$
$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	See previous error.
$(x+a)^n \neq x^n+a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+2$ $(2x+2)^2 = 4x^2+8x+4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parenthesis!
$\sqrt{-x^2+a^2} \neq -\sqrt{x^2+a^2}$	$\sqrt{-x^2+a^2} = (-x^2+a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$

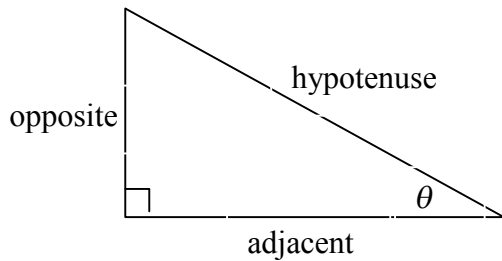
# Trigonometry

## Definition of the Trig Functions

### Right triangle definition

For this definition we assume that

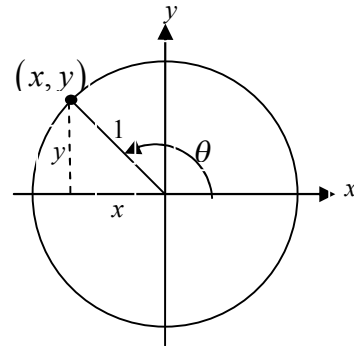
$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$



$$\begin{aligned} \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}} \end{aligned}$$

### Unit circle definition

For this definition  $\theta$  is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{1} = y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= \frac{x}{1} = x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

## Facts and Properties

### Domain

The domain is all the values of  $\theta$  that can be plugged into the function.

$$\begin{aligned} \sin \theta, & \quad \theta \text{ can be any angle} \\ \cos \theta, & \quad \theta \text{ can be any angle} \\ \tan \theta, & \quad \theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \csc \theta, & \quad \theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \sec \theta, & \quad \theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \cot \theta, & \quad \theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \end{aligned}$$

### Range

The range is all possible values to get out of the function.

$$\begin{aligned} -1 \leq \sin \theta \leq 1 & \quad \csc \theta \geq 1 \text{ and } \csc \theta \leq -1 \\ -1 \leq \cos \theta \leq 1 & \quad \sec \theta \geq 1 \text{ and } \sec \theta \leq -1 \\ -\infty < \tan \theta < \infty & \quad -\infty < \cot \theta < \infty \end{aligned}$$

### Period

The period of a function is the number,  $T$ , such that  $f(\theta + T) = f(\theta)$ . So, if  $\omega$  is a fixed number and  $\theta$  is any angle we have the following periods.

$$\begin{aligned} \sin(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cos(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \tan(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \\ \csc(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \sec(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cot(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \end{aligned}$$

## Formulas and Identities

### Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sin \theta = \frac{1}{\csc \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \cos \theta = \frac{1}{\sec \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Even/Odd Formulas

$$\sin(-\theta) = -\sin \theta \quad \csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta \quad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \quad \cot(-\theta) = -\cot \theta$$

### Periodic Formulas

If  $n$  is an integer.

$$\sin(\theta + 2\pi n) = \sin \theta \quad \csc(\theta + 2\pi n) = \csc \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta \quad \sec(\theta + 2\pi n) = \sec \theta$$

$$\tan(\theta + \pi n) = \tan \theta \quad \cot(\theta + \pi n) = \cot \theta$$

### Double Angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Degrees to Radians Formulas

If  $x$  is an angle in degrees and  $t$  is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

### Half Angle Formulas

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

### Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

### Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

### Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

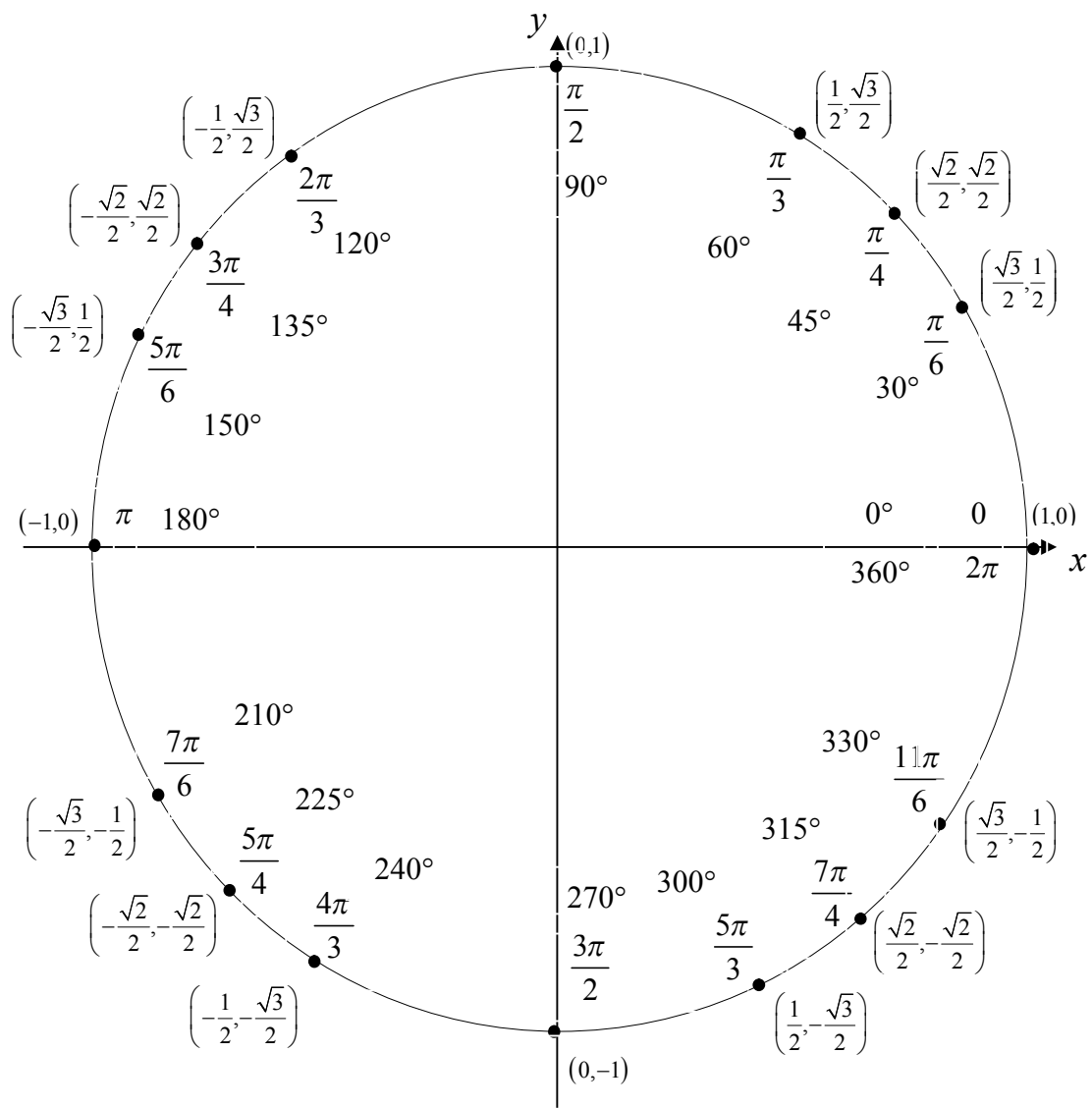
### Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

## Unit Circle



For any ordered pair on the unit circle  $(x, y)$  :  $\cos \theta = x$  and  $\sin \theta = y$

### Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

## Inverse Trig Functions

### Definition

$y = \sin^{-1} x$  is equivalent to  $x = \sin y$

$y = \cos^{-1} x$  is equivalent to  $x = \cos y$

$y = \tan^{-1} x$  is equivalent to  $x = \tan y$

### Inverse Properties

$$\cos(\cos^{-1}(x)) = x \quad \cos^{-1}(\cos(\theta)) = \theta$$

$$\sin(\sin^{-1}(x)) = x \quad \sin^{-1}(\sin(\theta)) = \theta$$

$$\tan(\tan^{-1}(x)) = x \quad \tan^{-1}(\tan(\theta)) = \theta$$

### Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

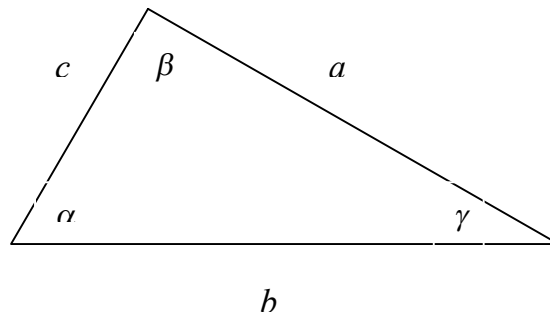
### Alternate Notation

$$\sin^{-1} x = \arcsin x$$

$$\cos^{-1} x = \arccos x$$

$$\tan^{-1} x = \arctan x$$

## Law of Sines, Cosines and Tangents



### Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

### Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

### Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2}\gamma}$$

### Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(\beta - \gamma)}{\tan \frac{1}{2}(\beta + \gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(\alpha - \gamma)}{\tan \frac{1}{2}(\alpha + \gamma)}$$