Mathematical Induction

The purpose of mathematical induction is to be able to prove an equation works for all natural numbers (1,2,3…) without actually calculating each one individually. This is done by making assumptions on what has been proved in the previous calculations. For example: If we have a line of dominoes and know that each one will knock the one in front of it down we only need to push the first one in order for all to fall. Similarly if we can prove that one number works in an equation, we can prove that the others following it can rely on that fact, just as the dominoes in the example.

The following three-step proof is an example of how to use mathematical induction to prove the following:

$$\sum_{1}^{n} x = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Step 1) Prove true for n = 1 by substituting a "1" everywhere there is an "n" in the original equation.

$$\sum_{1}^{1} x = 1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

Step 2) Assume true for n = k, "k" representing <u>*some*</u> natural number and "n" representing <u>*all*</u> possible natural numbers. This is done by substituting a "k" everywhere there is an "n" in the original equation.

$$\sum_{1}^{k} x = 1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$$

Step 3) Prove true for n=k+1, which would mean:

$$\sum_{1}^{k+1} x = 1 + 2 + 3 + \cdots + k + (k+1) = \frac{(k+1)[(k+1)+1]}{2}$$

From step 2 we know that $1 + 2 + 3 + \cdots + k = \sum_{1}^{k} x = \frac{k(k+1)}{2}$. By substituting this into the previous equation we have

$$\sum_{1}^{k+1} x = \sum_{1}^{k} x + (k+1) = \frac{(k+1)[(k+1)+1]}{2}$$

$$\sum_{1}^{k+1} x = \frac{k(k+1)}{2} + (k+1) = \frac{(k+1)[(k+1)+1]}{2}$$

$$\sum_{1}^{k+1} x = \frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)[(k+1)+1]}{2}$$

$$\sum_{1}^{k+1} x = \frac{k^2 + k + 2k + 2}{2} = \frac{(k+1)[(k+1)+1]}{2}$$

$$\sum_{1}^{k+1} x = \frac{k^2 + 3k + 2}{2} = \frac{(k+1)[(k+1)+1]}{2}$$

$$\sum_{1}^{k+1} x = \frac{(k+1)(k+2)}{2} = \frac{(k+1)[(k+1)+1]}{2}$$

$$\sum_{1}^{k+1} x = \frac{(k+1)[(k+1)+1]}{2} = \frac{(k+1)[(k+1)+1]}{2}$$

Once both sides of the equation are equal the proof is complete.