Graphing Rational Functions

A. Test to see if the graph has symmetry by plugging in (-x) in the function.

Options:
If the signs all stay the same or all change, f(-x) = f(x), then you have **even or y-axis symmetry**.

$$f(x) = \frac{x^2+3}{x^2-2}$$

$$f(-x) = \frac{(-x)^2+3}{(-x)^2-2}$$

$$f(-x) = \frac{x^2+3}{x^2-2}$$

If either the numerator or the denominator changes signs completely, f(-x)= -f(x) then you have **odd, or origin symmetry**.

$$f(x) = \frac{x^2}{x^3-x}$$

$$f(-x) = \frac{(-x)^2}{(-x)^3 - (-x)}$$

$$= \frac{x^2}{-x^3+x}$$

$$= -\left(\frac{x^2}{x^3-x}\right)$$

If neither of the above, then there is no symmetry.

$$f(x) = \frac{x^3+3}{x^2-4x}$$

$$f(-x) = \frac{(-x)^3+3}{(-x)^2-4(-x)}$$

$= \frac{-x^3+3}{x^2+4x}$

B. Test to find y-intercepts by replacing x with 0.

$$f(x) = \frac{3x^2+8}{x^3-2}$$

$$f(0) = \frac{3(0)^2+8}{(0)^3-2}$$

$$= \frac{8}{-2} = -4$$

$$\text{y-int. } (0,-4)$$

C. Test to find x-intercepts by setting the numerator equal to 0.
Shortcut: Since multiplying by the denominator will eliminate it, you can just set the numerator equal to zero.

$$f(x) = \frac{2x}{x+1}$$

$$0 = 2x \ \rightarrow 0 = x$$

$$\text{x-int. } (0,0)$$

D. Find the vertical asymptote by setting the denominator equal to zero.
The result is the equation of the vertical asymptote. Keep an eye out for holes. Holes occur when there is a factor that is the same both in the numerator and in the denominator. IT IS NOT A VERTICAL ASYMPTOTE because it simplifies away.

$$f(x) = \frac{x^2-9}{(x-3)(x+5)}$$

$$= \frac{(x-3)(x+3)}{(x-3)(x+5)}$$

$$x-3=0, \qquad x=3$$

Therefore at x=3 there is a hole.

$$x + 5 = 0 \; x = -5$$

Therefore at x=-5 there is a vertical asymptote.

E. Find the horizontal asymptote.
To find the horizontal asymptote you compare the degrees of the numerator and the denominator. Note: horizontal asymptotes can be crossed while vertical asymptotes can never be crossed.

Options:
If the degrees are the same, you take the ratio of the leading coefficients and that is your asymptote.

$$f(x) = \frac{5x^2}{x^2 + 7}$$

$$\text{ratio} = \frac{5}{1} \text{ so the asymptote is at } y = 5$$

If the degree of the numerator is less than the degree of the denominator, y=0 is your asymptote.

$$f(x) = \frac{4x - 9}{x^2 + 3}$$

$$1 < 2 \text{ so the asymptote is at } y = 0$$

If the degree of the numerator is exactly one greater than the degree of the denominator, there is no horizontal asymptote but there is a slant asymptote.

$$f(x) = \frac{x^2 + 2}{x}$$

$$2 > 1 \text{ so there is no horizontal asymptote}$$

F. Find the slant asymptote.
To have a slant asymptote, the degree of the numerator must be exactly one more than the degree of the denominator. To find the equations of the slant asymptote, divide the denominator into the numerator. If there is a slant asymptote, there will not be a vertical asymptote.

$$f(x) = \frac{x^2 + 2}{x}$$
$$2 > 1$$

$$\begin{array}{r} x \\ x\overline{)x^2 + 0x + 2} \\ -x^2 \\ \\ 2 \end{array}$$

y=x is the slant asymptote

G. Find points.
Plot at least one point between and beyond each x-intercept and vertical asymptote. Plug a number in for x and solve for y. You want to do this so you can find which side of the asymptotes you are going to graph on.

H. Graph using the information you have found.

Example 1:

$$f(x) = \frac{4x}{x-2}$$

A. Look for symmetry. $\frac{4(-x)}{(-x)-2}$

Since the numerator changed signs completely, but the denominator half changed there is no symmetry.

B. Find y-intercepts, if any. $f(x) = \frac{4(0)}{0-2} = \frac{0}{2} = 0$

The y-intercept is (0,0).

C. Find any x-intercepts. 0=4x 0=x

(0,0) is our x-intercept as well.

D. Find vertical asymptote(s). x-2=0

So x=2 is the vertical asymptote.

E. Find horizontal asymptote(s). 1=1

The asymptote is the ratio of 4/1, so the asymptote is y=4.

F. Find slant asymptote, if any. 1=1

Since the degrees are the same, there is no slant asymptote.

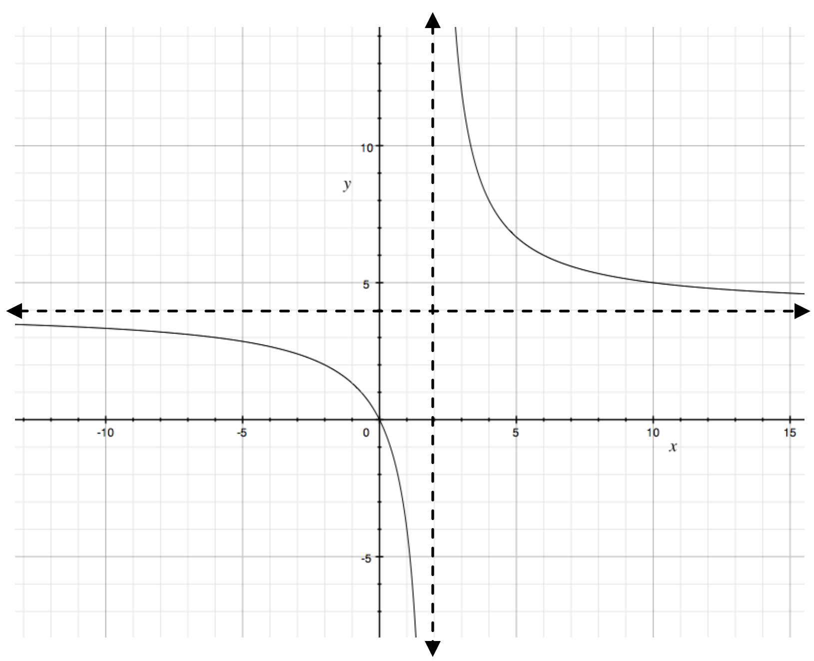
G & H. Find points and graph.

Found points:

(1, -4)

(3, 12)

(-1, 4/3)



Example 2:

$$f(x) = \frac{2x}{x^2 - 4}$$

A. Look for symmetry. $\frac{2(-x)}{(-x)^2-4} = \frac{-2x}{x^2-4} = -\left(\frac{2x}{x^2-4}\right)$

Since the numerator changed signs completely, but the denominator stayed the same, causing the function to become –f(x), there is odd symmetry.

B. Find y-intercepts, if any. $\frac{2(0)}{(0)^2-4} = \frac{0}{-4} = 0$

(0,0) is the y-intercept.

C. Find any x-intercepts. $\frac{0}{2} = \frac{2x}{2}$ 0=x

(0,0) is also the x-intercept.

D. Find vertical asymptote(s).

$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$\sqrt{x^2} = \pm\sqrt{4}$$

X = ±2

X=-2 (-2,0) and x=2 (2,0) are both vertical asymptotes.

E. Find horizontal asymptote(s). 1<2

Y=0 is the horizontal asymptote.

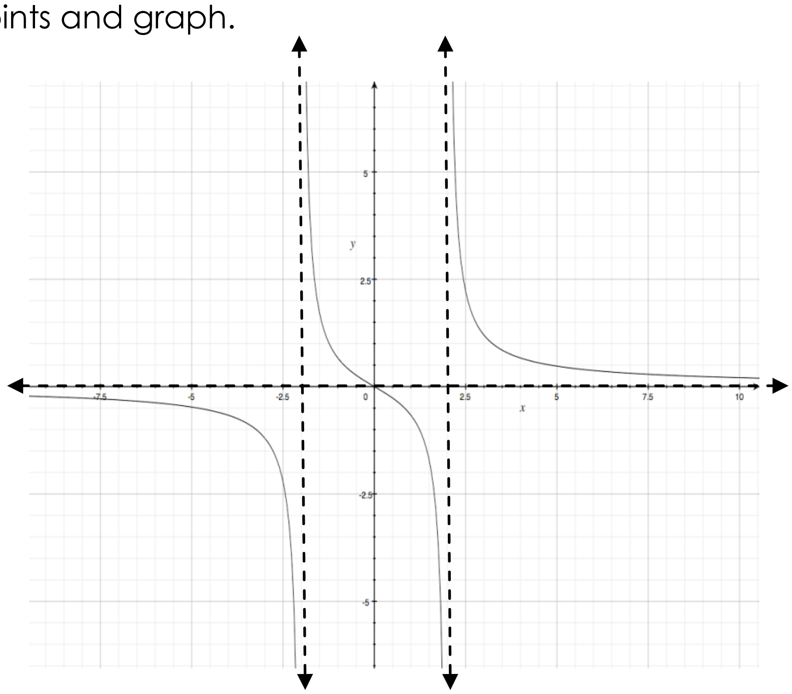
F. Find slant asymptote, if any. 1<2

Since the numerator degree is smaller than the denominator degree, there is no slant asymptote.

G & H. Find points and graph.

Found points:

(-1, 2/3)
(1, -2/3)
(3, 1.2)
(-3, -1.2)



Example 3:

$$f(x) = \frac{x^2 - 1}{x}$$

A. Look for symmetry. $\frac{(-x)^2-1}{(-x)} = \frac{x^2-1}{-x} = -\left(\frac{x^2-1}{x}\right)$

Since the denominator changed signs completely, but the numerator stayed the same, causing the function to become –f(x), there is odd symmetry.

B. Find y-intercepts, if any. $\frac{0^2-1}{0} = \frac{-1}{0}$

This is undefined so that means there is no y-intercept.

C. Find any x-intercepts.

$x^2 - 1 = 0$

$x^2 = 1$

$\sqrt{x^2} = \pm\sqrt{1}$

X= ±1

X=-1 (-1,0) and x=1 (1,0) are both vertical asymptotes.

D. Find vertical asymptote(s). x=0

X=0 is our vertical asymptote.

E. Find horizontal asymptote(s). 2>1

Because the numerator degree is one more than then denominator degree, there is no horizontal asymptote.

F. Find slant asymptote, if any. 2>1

Yes there is a slant asymptote.

X

x)$\overline{x^2 + 0x - 1}$

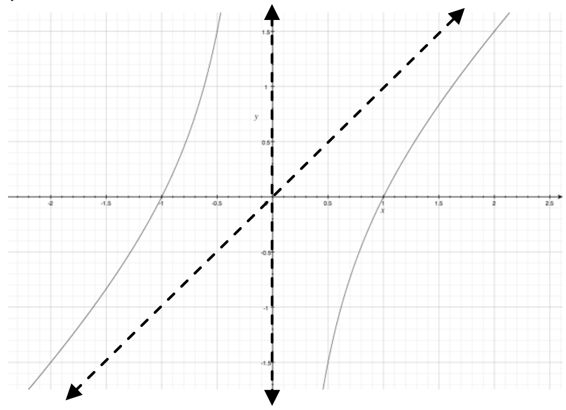
$-x^2$

-1

y=x is the slant asymptote

G & H. Find points and graph.

(1.5, 0.83) & (-1.5, -0.83)



Example 4:

$$f(x) = \frac{x^2-2x-3}{x^2-8x+15} \qquad \frac{(x-3)(x+1)}{(x-3)(x-5)}$$

A. Look for symmetry. $\frac{(-x)^2-2(-x)-3}{(-x)^2-8(-x)+15} = \frac{x^2+2x-3}{x^2+8x-15}$

Since both the numerator and the denominator half changed, there is no symmetry.

B. Find y-intercepts, if any. $\frac{(0)^2-2(0)-3}{(0)^2-8(0)+15} = \frac{-3}{15} = -\frac{1}{5}$

(0, -1/5) is the y-intercept.

C. Find any x-intercepts. X+1=0

X=-1

Since(x-3) cancels away, it is not an x-intercept.

X=-1 (-1,0) is the x-intercept.

D. Find vertical asymptote(s). $\frac{(x-3)(x+1)}{(x-3)(x-5)}$

x-3=0 x-5=0

x=3 x=5

Because (x-3) is in both the denominator and the numerator it can cancel out, creating a hole at x=3. At x=5 there is a vertical asymptote.

E. Find horizontal asymptote(s). 2=2

The asymptote is the ratio of 1/1, so the asymptote is y=1.

F. Find slant asymptote, if any. 2=2

Since the degrees are the same, there is no slant asymptote.

G & H. Find points and graph.

Found point:

(6, 7)