Graphing Rational Functions

A. Test to see if the graph has symmetry by plugging in (-x) in the function.

Options:
If the signs all stay the same or all change, f(-x) = f(x), then you have **even or y-axis symmetry**.

$$f(x) = \frac{x^2+3}{x^2-2}$$

$$f(-x) = \frac{(-x)^2+3}{(-x)^2-2}$$

$$f(-x) = \frac{x^2+3}{x^2-2}$$

If either the numerator or the denominator changes signs completely, f(-x)= -f(x) then you have **odd, or origin symmetry**.

$$f(x) = \frac{x^2}{x^3-x}$$

$$f(-x) = \frac{(-x)^2}{(-x)^3 - (-x)}$$

$$= \frac{x^2}{-x^3+x}$$

$$= -\left(\frac{x^2}{x^3-x}\right)$$

If neither of the above, then there is no symmetry.

$$f(x) = \frac{x^3+3}{x^2-4x}$$

$$f(-x) = \frac{(-x)^3+3}{(-x)^2-4(-x)}$$

$$= \frac{-x^3+3}{x^2+4x}$$

B. Test to find y-intercepts by replacing x with 0.

$$f(x) = \frac{3x^2+8}{x^3-2}$$

$$f(0) = \frac{3(0)^2+8}{(0)^3-2}$$

$$= \frac{8}{-2} = -4$$

$$\text{y-int. } (0, -4)$$

C. Test to find x-intercepts by setting the numerator equal to 0.
Shortcut: Since multiplying by the denominator will eliminate it, you can just set the numerator equal to zero.

$$f(x) = \frac{2x}{x+1}$$

$$0 = 2x \rightarrow 0 = x$$

$$\text{x-int. } (0, 0)$$

D. Find the vertical asymptote by setting the denominator equal to zero.
The result is the equation of the vertical asymptote. Keep an eye out for holes. Holes occur when there is a factor that is the same both in the numerator and in the denominator. IT IS NOT A VERTICAL ASYMPTOTE because it simplifies away.

$$f(x) = \frac{x^2-9}{(x-3)(x+5)}$$

$$= \frac{(x-3)(x+3)}{(x-3)(x+5)}$$

$$x - 3 = 0, \qquad x = 3$$

Therefore at x=3 there is a hole.

$$x + 5 = 0 \; x = -5$$

Therefore at x=-5 there is a vertical asymptote.

E. Find the horizontal asymptote.
To find the horizontal asymptote you compare the degrees of the numerator and the denominator. Note: horizontal asymptotes can be crossed while vertical asymptotes can never be crossed.

Options:
If the degrees are the same, you take the ratio of the leading coefficients and that is your asymptote.

$$f(x) = \frac{5x^2}{x^2 + 7}$$

$$\text{ratio} = \frac{5}{1} \text{ so the asymptote is at } y = 5$$

If the degree of the numerator is less than the degree of the denominator, y=0 is your asymptote.

$$f(x) = \frac{4x - 9}{x^2 + 3}$$

$$1 < 2 \text{ so the asymptote is at } y = 0$$

If the degree of the numerator is exactly one greater than the degree of the denominator, there is no horizontal asymptote but there is a slant asymptote.

$$f(x) = \frac{x^2 + 2}{x}$$

$$2 > 1 \text{ so there is no horizontal asymptote}$$

F. Find the slant asymptote.
To have a slant asymptote, the degree of the numerator must be exactly one more than the degree of the denominator. To find the equations of the slant asymptote, divide the denominator into the numerator. If there is a slant asymptote, there will not be a vertical asymptote.

$$f(x) = \frac{x^2 + 2}{x}$$

$$2 > 1$$

$$\begin{array}{r} x \\ x\overline{)x^2 + 0x + 2} \\ -x^2 \\ \\ 2 \end{array}$$

y=x is the slant asymptote

G. Find points.
Plot at least one point between and beyond each x-intercept and vertical asymptote. Plug a number in for x and solve for y. You want to do this so you can find which side of the asymptotes you are going to graph on.

H. Graph using the information you have found.