Example 1:

\[ f(x) = \frac{4x}{x - 2} \]

A. Look for symmetry.

Since the numerator changed signs completely, but the denominator half changed there is no symmetry.

B. Find y-intercepts, if any.

\[ f(x) = \frac{4(0)}{0-2} = \frac{0}{-2} = 0 \]

The y-intercept is (0,0).

C. Find any x-intercepts.

\[ 0=4x \quad 0=x \]

(0,0) is our x-intercept as well.

D. Find vertical asymptote(s).

\[ x-2=0 \]

So x=2 is the vertical asymptote.

E. Find horizontal asymptote(s).

\[ 1=1 \]

The asymptote is the ratio of 4/1, so the asymptote is y=4.

F. Find slant asymptote, if any.

\[ 1=1 \]

Since the degrees are the same, there is no slant asymptote.

G & H. Find points and graph.

Found points:

(1, -4)
(3, 12)
(-1, 4/3)
Example 2:

\[ f(x) = \frac{2x}{x^2 - 4} \]

A. Look for symmetry.

\[
\frac{2(-x)}{(-x)^2 - 4} = \frac{-2x}{x^2 - 4} = -\left( \frac{2x}{x^2 - 4} \right)
\]

Since the numerator changed signs completely, but the denominator stayed the same, causing the function to become \(-f(x)\), there is odd symmetry.

B. Find y-intercepts, if any.

\[
\frac{2(0)}{(0)^2 - 4} = \frac{0}{-4} = 0
\]

(0,0) is the y-intercept.

C. Find any x-intercepts.

\[
0 = \frac{2x}{2} \quad 0=x
\]

(0,0) is also the x-intercept.

D. Find vertical asymptote(s).

\[
x^2 - 4 = 0
\]

\[
x^2 = 4
\]

\[
\sqrt{x^2} = \pm\sqrt{4}
\]

\[
x = \pm 2
\]

X=-2 (-2,0) and x=2 (2,0) are both vertical asymptotes.

E. Find horizontal asymptote(s).

\[
1<2
\]

Y=0 is the horizontal asymptote.

F. Find slant asymptote, if any.

\[
1<2
\]

Since the numerator degree is smaller than the denominator degree, there is no slant asymptote.

G & H. Find points and graph.

Found points:

(-1, 2/3)
(1, -2/3)
(3, 1.2)
(-3, -1.2)
Example 3:

\[ f(x) = \frac{x^2 - 1}{x} \]

A. Look for symmetry.

\[ \frac{(-x)^2 - 1}{-x} = \frac{x^2 - 1}{-x} = -\left(\frac{x^2 - 1}{x}\right) \]

Since the denominator changed signs completely, but the numerator stayed the same, causing the function to become \(-f(x)\), there is odd symmetry.

B. Find y-intercepts, if any.

\[ \frac{0^2 - 1}{0} = \frac{-1}{0} \]

This is undefined so that means there is no y-intercept.

C. Find any x-intercepts.

\[ x^2 - 1 = 0 \]
\[ x^2 = 1 \]
\[ \sqrt{x^2} = \pm \sqrt{1} \]
\[ x = \pm 1 \]

X = -1 (-1,0) and x=1 (1,0) are both vertical asymptotes.

D. Find vertical asymptote(s).

X=0 is our vertical asymptote.

E. Find horizontal asymptote(s).

Because the numerator degree is one more than then denominator degree, there is no horizontal asymptote.

F. Find slant asymptote, if any.

\[ 2 > 1 \]

Yes there is a slant asymptote.

\[ \frac{x^2 + 0x - 1}{-x^2} \]

\[ y=x \text { is the slant asymptote} \]

G & H. Find points and graph.

(1.5, 0.83) & (-1.5, -0.83)
Example 4:

\[ f(x) = \frac{x^2-2x-3}{x^2-8x+15} \quad \frac{(x-3)(x+1)}{(x-3)(x-5)} \]

A. Look for symmetry.

\[
\frac{(-x)^2-2(-x)-3}{(-x)^2-8(-x)+15} = \frac{x^2+2x-3}{x^2+8x-15}
\]

Since both the numerator and the denominator half changed, there is no symmetry.

B. Find y-intercepts, if any.

\[
\frac{(0)^2-2(0)-3}{(0)^2-8(0)+15} = \frac{-3}{15} = -\frac{1}{5}
\]

(0, -1/5) is the y-intercept.

C. Find any x-intercepts.

\[ x+1=0 \]
\[ x=-1 \]

Since (x-3) cancels away, it is not an x-intercept.

\[ x=-1 \ (-1,0) \ is \ the \ x-intercept. \]

D. Find vertical asymptote(s).

\[ \frac{(x-3)(x+1)}{(x-3)(x-5)} \]
\[ x-3=0 \quad x-5=0 \]
\[ x=3 \quad x=5 \]

Because (x-3) is in both the denominator and the numerator it can cancel out, creating a hole at x=3. At x=5 there is a vertical asymptote.

E. Find horizontal asymptote(s).

\[ \frac{2}{2} = 1 \]

The asymptote is the ratio of 1/1, so the asymptote is y=1.

F. Find slant asymptote, if any.

Since the degrees are the same, there is no slant asymptote.

G & H. Find points and graph.

Found point:

(6, 7)