Math Department Fall Seminar

Wednesday, Nov 9, 2011
1:30pm Room 518

Speaker: **T.H. Steele**

Title: **Continuity and chaos in discrete dynamical systems**

Abstract:

In the latter part of the nineteenth century, there was a belief in the deterministic, clockwork precision of the universe. From this belief arose an interest in establishing the stability of the planetary motions in our solar system. Oscar II, King of Sweden and Norway, initiated a mathematical competition in 1887 to celebrate his sixtieth birthday in 1889. One of the problems, posed by Karl Weierstrass, dealt with this stability:"Given a system of arbitrarily many mass points that attract each other according to Newton's laws, assuming that no two points ever collide, give the coordinates of the individual points for all time…."

Henri Poincare attacked this problem for our solar system but found this too complicated to solve. He switched to a three body problem and instead studied the discrete time trajectories of the point masses. From his analysis Poincare submitted a solution to the three body problem. He was awarded the prize but in the publication review of the work, the referee Phragmen found an error. Poincare came to understand that a concept we now call

sensitive dependence on initial conditions was involved, rendering impossible the long term prediction of the three body problem.

Since Poincare's seminal work, chaotic and dynamical systems have been used to model many systems in the physical and life sciences. Typically, these problems involve a continuous function f which maps a compact set X into itself, and study the dynamics of the sequence $\{x, f(x), f(f(x)), f(f(f(x))) \ldots\}$, where x is an element of X. For each x in X, one also can define $\omega(x, f)$, the ω-limit set of f at x, as the cluster set of the sequence $\{x, f(x), f(f(x)), f(f(f(x))) \ldots\}$. In the first part of this talk we focus our attention on how ω-limit sets $\omega(x, f)$ are affected by perturbations in the initial condition x, the generating function f, or both. We then turn our attention to a description of the typical behavior for a function f on one of its ω-limit sets $\omega(x, f)$.

While we cannot solve the problem posed by Weierstrass, our results will allow us to give a very specific description of our solar system's very likely long term behavior.

Everyone is Welcome.